Frequency Dependence in a Real-Time Monetary Policy Rule

Richard Ashley — **Virginia Tech**
Kwok Ping Tsang — **Virginia Tech**

Randal J. Verbrugge
Federal Reserve Bank of Cleveland*

August 4, 2017

Abstract

We estimate a monetary policy rule for the US allowing for possible frequency dependence – i.e., allowing the central bank to respond differently to more persistent innovations than to more transitory innovations, in both the unemployment rate and the inflation rate. Our estimation method uses real-time data in these rates – as did the FOMC – and requires no *a priori* assumptions on the pattern of frequency dependence or on the nature of the processes generating either the data or the natural rate of unemployment. Unlike other approaches, our estimation method allows for possible feedback in the relationship. Our results convincingly reject linearity in the monetary policy rule, in the sense that we find strong evidence for frequency dependence in the key coefficients of the central bank's policy rule: i.e., the central bank's federal funds rate response to a fluctuation in either the unemployment or the inflation rate depended strongly on the persistence of this fluctuation in the recently observed (real-time) data. These results also provide useful insights into how the central bank's monetary policy rule has varied between the Martin-Burns-Miller and the Volcker-Greenspan time periods.

JEL Codes: E52, C22, C32

Keywords: Taylor rule, frequency dependence, spectral regression, real-time data

*The authors would like to thank the editor and two anonymous referees for helpful comments. Ashley: Department of Economics, Virginia Tech, Pamplin Hall (0316), Blacksburg, VA 24061-0316. Email: ashleyr@vt.edu. Tsang (Corresponding author): Department of Economics, Virginia Tech, Pamplin Hall (0316), Blacksburg, Virginia, 24061. Email: byront@vt.edu. Verbrugge: Research Department, Federal Reserve Bank of Cleveland P.O. Box 6387 Cleveland, Ohio 44101-1387. Email: Randal.Verbrugge@clev.frb.org. The views stated herein are those of the authors and are not necessarily those of the Federal Reserve Bank of Cleveland or of the Federal Reserve System.

1 Introduction

1.1 Background

Following Taylor (1993), there has been an intense focus on Taylor-type monetary policy rules, such as:

$$i_t = \alpha + \phi_\pi \pi_t + \phi_u u_t + e_t \tag{1}$$

where i_t is the federal funds rate, π_t is the annualized inflation rate from period $t-1$ to period t, u_t is a measure of real activity (output gap or unemployment rate) in period t, and e_t is a stationary exogenous monetary shock. There are many variants of Equation (1). Theory often suggests forward-looking versions (e.g. Clarida, Gali and Gertler (2000)); real-time lags in data collection motivate the use of lagged inflation and real activity, i.e., a backward-looking monetary policy rule (e.g. McCallum (1997)); there is increasing focus on time-variation in the natural rate of interest (e.g., Laubach and Williams 2003, Curdia et al. 2015, Hamilton et al. 2015, and Holston, Laubach and Williams 2016); and interest rate smoothing considerations (as well as the statistical properties of i_t) motivate adding lags of i_t to the right-hand side of (1).[1] A number of studies have used variants of Equation (1) to conclude that the central bank's policy changed markedly starting with Volcker; see, e.g. Clarida, Gali and Gertler (2000) and Judd and Rudebusch (1998).[2]

Taylor originally developed this monetary policy rule as a descriptive device. More recently, a Taylor-type rule (with appropriate arguments and parameter values) is found to be optimal in some dynamic New Keynesian macroeconomic models – see Woodford (2003) or Gali (2009) for a textbook treatment. Decades ago, however, Friedman (1968) and Phelps (1968) introduced the concept of a natural rate of unemployment; and a related literature on the Phillips curve argues

[1] Consolo and Favero (2009) argues, in the forward-looking context, that the inertia is an artifact of a weak-instrument problem for expected inflation. Forward-looking monetary policy rules are not considered here, because doing so would require a substantial number of valid instruments for π_{t+j} and u_{t+j}, and lagged values do not provide useful instruments due to the identification problem discussed by Jondeau, Bihan and Galles (2004). Rudebusch (2002) disagrees with the interest rate smoothing interpretation; using evidence from the term structure, he shows that monetary policy inertia is more likely due to persistent shocks that the central bank faces.

[2] Using the the time-varying parameter framework, Cogley and Sargent (2005), Kim and Nelson (2006) and McCulloch (2007) come to a similar conclusion. However, Sims (2001) and Sims and Zha (2006) find that there is less evidence for significant changes in the reaction coefficients ϕ if one allows for time-varying variance in the monetary policy shock.

for the existence of a time-varying non-accelerating inflation rate of unemployment (NAIRU). If there is a time-varying NAIRU, it implies that not all unemployment rate movements are economically equivalent. A reasonable central bank will thus respond differently to natural rate movements than to fluctuations about the natural rate. Hence, a linear policy rule such as Equation (1) – with u_t as an argument and with a constant value for ϕ_u – must be seriously misspecified.

A fairly common means of addressing this issue is to use an "unemployment gap" for u_t in Equation (1), as (for example) in McCulloch (2007).[3] To calculate the unemployment gap, one needs an explicit estimate of the time-varying "long-run normal" level of unemployment. Such estimates are inherently problematic in that they are usually estimated very imprecisely, are subject to large revisions, and typically hinge upon untested (and perhaps untestable) auxiliary assumptions about the natural rate data generating process – such as an explicit formulation of its persistence, which may well be incorrect. They are also generally based upon two-sided filters, which (as noted below) are known to distort the dynamics of time series relationships. The estimates can also vary across concepts and methods widely (see Tasci and Verbrugge (2014)). Use of an output gap instead of an unemployment gap in Equation (1), also common, does not improve matters.

Much of this is well-understood by practitioners. What is somewhat less well-understood is that a similar set of issues arises with respect to the inflation terms in policy rules. In particular, central banks likely respond differently to more persistent innovations in inflation than to less persistent innovations. A crucial issue in inflation measurement is noise: in the short run, inflation is often subject to large transitory influences, which can affect an aggregate price index for long periods. Policymakers have argued – e.g. Mishkin (2007) – that the central bank should not respond to transitory fluctuations in inflation. If the assumed policy rule does not take this differential response into account, Equation (1) will again be seriously misspecified. Some analysts have attempted to address this issue by making use of "core inflation" measures in Equation (1). This expedient is valid, however, only if all movements in the core inflation measure are identically persistent, which is emphatically not the case; for example, see Bryan and

[3]See Orphanides (2008) and Knotek et al. (2016) for a more general discussion. Other notable studies, such as Ball and Tchaidze (2002) and Detmeister and Babb (2017), do not use an unemployment gap specification.

Meyer (2002) and Dolmas and Wynne (2008). Note that this issue is distinct from whether π_t should instead enter Equation (2) in terms of an "inflation gap," where this gap is the difference between π_t and an inflation target, π_t^*.

Related issues arise in two prominent recent studies that use variants of Equation (1) to understand monetary policy behavior and its evolution over time. First, Orphanides (2002) uses real-time data and shows that the Federal Open Market Committee's (FOMC) forecast of inflation is biased during the Great Inflation period. He finds that the Federal Reserve actually reacts aggressively to its biased forecast; but standard analysis using *ex post* data leads to the conclusion that the FOMC only responds weakly to inflation.

Second, Ball and Tchaidze (2002) investigate the claim of Blinder and Yellen (2001) that the FOMC, under the guidance of Alan Greenspan, behaved differently in the late 1990s than previously, displaying "forbearance" - i.e., holding interest rates steady under conditions that would ordinarily have triggered a tightening. In Ball and Tchaidze's baseline specification, which is Equation (1), the federal funds rate is related to the inflation rate and the unemployment rate. Their estimated policy reaction coefficients vary markedly between the 'old economy' period, pre-1996, and the 'new economy' period, 1996-2000. During the 'old economy' period, their estimated policy rule displays a strong relationship to both inflation and the unemployment rate. But during the 'new economy' period, their estimates point to a far more muted response to unemployment rate fluctuations. Ball and Tchaidze (2002) then argue that these results are explained by an implicit assumption of a fixed NAIRU in the baseline policy rule specification. Upon re-specifying the relationship to include a time-varying NAIRU taken from other studies, they find that their coefficient estimates are not very different across the two periods. We re-visit this issue in Section 2.4 below, exploiting the fact that our approach gracefully allows for a time-varying natural rate of unemployment with no need to separately estimate its variation or import estimates from another study.[4]

[4]See Tasci and Verbrugge (2014) for a recent discussion of competing "natural rate" concepts and associated estimates.

1.2 The Present Paper

In this paper, we estimate the central bank's assumed monetary policy rule using a new method proposed by Ashley and Verbrugge (2009). The Technical Appendix (Section 4 below) describes the latest version of this method. We use it to investigate the possibility the the policy rule reacts in different ways to fluctuations of differing persistence in the real-time data. In particular, this method allows us to estimate whether (and how) the central bank differentially responds in real time to perceived changes in either the unemployment rate or the inflation rate, with persistence levels varying in steps from permanent ("zero-frequency") to completely transitory ("high-frequency"). We use a moving window of length 36 months in the analysis of the inflation rate data, which implies that persistence can vary in 19 steps, from completely "permanent" – referring to a fluctuation with an average reversion period of more than 36 months – to "temporary," referring to a fluctuation with an average reversion period of just two months.[5] For reasons explained in Section 2.1 below, we use a considerably longer moving window – 120 months in length – in decomposing the unemployment rate data into frequency (persistence) components; explicitly allowing for varying persistence in the central bank's responses to fluctuations in the unemployment rate renders explicit modeling of a time-varying NAIRU unnecessary.

Because a moving window on the data set is used, our method is gracefully compatible with the real-time data on inflation and unemployment rates which are actually available to the central bank policymakers whose behavior is being modelled. Moreover, because our frequency decomposition of the data in the window for time t uses only the real-time data for that rate actually available at time t, our partitioning of each rate into persistence (or frequency) components is, by construction, a backward-looking ("one-sided") filtering. Two-sided filters – e.g., those applied to both the explanatory and dependent variables in classical RBC studies or in papers such as Cochrane (1989) – distort relationships amongst variables which are in feedback with one another, because two-sided filtering inherently mixes up future and past values of the

[5] The Technical Appendix describes how the real-time data on the inflation rate and the unemployment rate are decomposed into frequency components which add up to the original data series. 'Reversion period' is intuitively defined as follows: a fluctuation which tends to self-reverse on a time scale shorter than the reversion period associated with a given frequency component will have little impact on this frequency component. Section 4.1 of the Appendix exposits this decomposition in detail; Section 4.3 motivates this concept using an explicit example with a short (10-month) window. Table 1 summarizes the 19 frequency components and reversion periods allowed by the 36-month window used for the inflation rate data decomposition here.

time series. (See Ashley and Verbrugge (2009) for a more detailed exposition of this point.[6]) Thus, our analysis provides consistent estimates of the frequency dependence in monetary policy rules even where – as one might expect – there is feedback between the federal funds rate and the inflation/unemployment explanatory variates in the policy rule.[7]

While the original Taylor-type monetary policy response function is attractive in its simplicity, our frequency-dependent extension of it broadens its generality and descriptive power, yielding novel results. More explicitly, we find that the FOMC responds differently to highly persistent innovations in the unemployment rate – which one might largely identify with natural rate fluctuations – than it does to more transitory innovations. Similarly, the central bank's responses to inflation-rate innovations are also frequency-dependent. These findings – that the policy response coefficients in the FOMC's policy response function were not actually constants, but instead depended on the persistence of fluctuations in the macroeconomic variables these coefficient are multiplying – imply that a model with constant coefficients (such as Equation (1) above) is seriously mis-specified and hence yields inconsistent (and misleadingly unstable) parameter estimates.

Appropriately allowing for frequency dependence in the monetary response coefficients yields a clearer picture of how the FOMC's actual policy rule has evolved over time – e.g., how it differs for the Martin-Burns-Miller (MBM) period (roughly March, 1960 to August, 1979) versus the Volcker-Greenspan-Bernanke (VGB) period, here taken to run from September, 1979 to August, 2008. In particular, estimates of Equation (1) as it stands imply that the FOMC was responsive to unemployment rate fluctuations in the MBM period but not in the VGB period; in contrast, our results in Section 2.3 show that the FOMC was significantly responsive to unemployment rate fluctuations in both periods once one allows for frequency dependence in the response coefficients. In addition – as noted above – in Section 2.4 we re-visit the Ball and Tchaidze (2002) result described at the end of Section 1.1 above. Allowing for frequency dependence in the monetary response coefficients, we find strong evidence that the FOMC's responses to both inflation and unemployment fluctuations were significant in both the "old economy" and

[6]For the same reason, applying a two-sided filter to each of two time series likewise distorts the crosscorrelations between them, even in the absence of feedback.

[7]For this reason – and because such calculations are incompatible with the use of real-time data – two-sided cross-spectral estimates are not quoted here.

the "new economy" periods, without any need to include a separately-estimated time varying NAIRU in the policy rule.[8]

We do not interpret our results as implying that that the FOMC explicitly decomposed fluctuations in the inflation and unemployment rates into different frequency or persistence levels, and then mechanically followed a Taylor-type rule of the form we estimate, although it is certainly arguable that these policymakers had something like this in mind.[9] Instead, the goal of the paper is to utilize a richer statistical model to describe the behavior of the central bank, allowing the data to better inform us as to the manner in which the central bank has responded to fluctuations in these macroeconomic variables.

2 Empirical Results

2.1 Data Description and Plan of This Section

We use real-time data on the unemployment rate (u_t) and the inflation rate (π_t) from St. Louis Federal Reserve Bank ALFRED data set, so that the data we are analyzing correspond closely to those which were available to the FOMC at the time it set the federal funds rate (i_t).[10] The federal funds rate itself is not revised; we use the monthly average of this variable.

More specifically, we use the civilian unemployment rate for u_t and we use the inflation rate defined as the 12-month growth rate in percentage terms – i.e., $100ln(CPI_t/CPI_{t-12})$ – where CPI_t is the non-seasonally adjusted Consumer Price Index for urban wage earners and clerical workers until February 1978 and the non-seasonally adjusted Consumer Price Index for all urban

[8]At the suggestion of a referee, we conducted a brief investigation into the post-sample forecasting effectiveness of our models, using explanatory variables partitioned into frequency components, relative to that of analogous models specified without frequency dependence. The results were encouraging, in that one or more of our disaggregated models is always somewhat superior on this metric, lending credence to our modeling approach. We note, however, that the present study is about inference rather than forecasting, and we lay no claim here to having developed a new forecasting approach.

[9]For example, Meyer, Venkatu and Zaman (2013) at the Federal Reserve Bank of Cleveland comment that: "By specifying the inflation threshold in terms of its forecasted values, the FOMC will still be able to 'look through' transitory price changes, like they did, for example, when energy prices spiked in 2008. At that time, the year-over-year growth rate in the Consumer Price Index (CPI) jumped up above 5.0 percent but subsequently plummeted below zero a year later when the bottom fell out on energy prices. At the time, the Committee maintained the federal funds rate target at 2.0 percent, choosing not to react to the energy price spike."

[10]Source: http://research.stlouisfed.org/fred2/. See Orphanides (2001) for evidence that estimated monetary policy rules are likely not robust to the vintage of the data.

consumers thereafter.[11] Owing to considerations related to our filtering, our regressions actually begin in January 1965. Our sample period ends in August 2008, just prior to the point when the sample variation in i_t becomes minimal.

Following Clarida, Gali, and Gertler (2000), we primarily consider two sub-sample periods. The first of these is January 1965 to August 1979, which roughly corresponds to the Martin-Burns-Miller period and is here denoted 'MBM'.[12] The second sub-sample runs from September 1979 to August 2008; it covers Volcker's, Greenspan's and part of Bernanke's tenures; it is hence here denoted denoted 'VGB'. Most of the VGB period is also referred to as the 'Great Moderation' – see McConnell and Perez-Quiros (2000) and Kim and Nelson (1999) – as it is characterized by low variance in most macroeconomic variables. Since the onset of the Great Recession, of course, most macroeconomic variables have become more volatile.

We also, in Section 2.4, estimate and compare models utilizing two sub-samples, one running from September 1987 to December 1994 and another running from January 1995 to December 2000. These dates were chosen so as to match up with the 'old economy' and 'new economy' periods originally defined (on the basis of productivity movements) in Ball and Tchaidze (2002). These results shed light on whether the FOMC's monetary policy rule in fact shifted between these two sub-periods, once we account for the frequency dependence in the policy rule.

Each observation on u_t and π_t is here decomposed into frequency (persistence) components, as described in the Technical Appendix (in Section 4.1), using moving windows. In specifying the length of the moving windows used in decomposing the real-time unemployment rate, we note that the natural rate of unemployment is thought to be a quite slowly-varying time series. For instance, in a robustness check exercise, Boivin (2006) uses a five-year moving average of the unemployment rate as a proxy for the natural rate. An exercise that estimated the width of a moving average of the unemployment rate that best matched prominent estimates of the NAIRU resulted in a moving average of nine years.[13] A shorter window - of length, say, 36 months - would risk including business cycle effects within the lowest-frequency unemployment rate component,

[11] As noted above, real-time values data are used, so the value for CPI_{t-12} in π_t is that which was available when CPI_t was released. Also, since inflation and unemployment data become available with a 1-month lag, we match the federal funds rate in each month with the data released in that month. For example, in April 2009 we use the inflation and unemployment data for March 2009.

[12] See Meltzer (2005) for a discussion of the high inflation rate during the latter part of this period.

[13] We are indebted to a referee who performed this analysis.

which here will extract the time variation in the NAIRU. Putting this differently, this window length choice would impose the restriction that the central bank responds in much the same way to an unemployment rate fluctuation with a reversion period of 36 months as it does to a fluctuations with substantially larger reversion periods – e.g., of 5 years, or even 10 years. Hence, we conjectured that a ten-year window would likely result in a satisfactory decomposition for the unemployment rate. As will be evident below, the outcome of the empirical analysis presented here will inform us as to whether or not a shorter window was adequate. In specifying the length of the moving windows used in decomposing the real-time inflation rate, in contrast, we considered that a central bank might very well react differently to a fluctuation in the real-time inflation rate which has persisted just 12 months than to one which has persisted ca. 36 months, but that it seems unlikely *a priori* that it would attend to a 36-month-long fluctuation substantially differently than to one which has persisted substantially longer than this.[14] We thus judged that a moving 36-month window would suffice for the inflation rate.[15]

These frequency components – for the real-time sample data on u_t and π_t – were used to obtain the empirical results for the paper, organized as follows:

- Section 2.2 Results for An Ordinary (But Dynamic) Taylor-type Policy Rule Model Ignoring Possible Frequency Dependence
- Section 2.3 Results for the Frequency Dependent Model

[14] A (centered) 36-month moving average is often used as an estimate of trend inflation; see, for example, Cecchetti (1997) or Brischetto and Richards (2007); Giannone and Matheson (2007) and Higgins and Verbrugge (2015) argue for using even shorter moving averages.

[15] Our empirical results are not particularly sensitive to specifying somewhat shorter (or longer) moving windows for use in decomposing u_t and π_t. The sample data within each window are filtered in order to extract the frequency (persistence) components used in the empirical analysis here; the details of how this filtering is done are described in the Technical Appendix below, Section 4. But it is worth at least mentioning at the present point that this filtering provides a usefully accurate decomposition for the last sample observation in a moving window only if the sample observations in the window are augmented by a number of 'projected' observations forecasting future values of the time series beyond the sample data used in (and decomposed via) the window. This projection is necessary so as to ameliorate the well-known 'edge effects' in the filters used. For this reason, after some experimentation in this regard, we find it both necessary and adequate to specify our 36-month 'data windows' for π_t to include the 30 (real-time) sample observations on the inflation (up to month t, as known in month t), augmented by 1-step-ahead through 6-step-ahead projections of π_t for the next six months, made using an AR(4) model estimated over these 30 sample observations. Similarly, our 120-month moving windows for decomposing u_t utilize the 96 months of sample data on the real time unemployment rate – up through month t, as known at month t – and the remaining 24 observations in the window consist of corresponding 1-step-ahead through 24-step-ahead AR(4) projections of the unemployment rate. These choices imply that roughly 80% of each window consists of sample data; as with the window lengths themselves, we find that our empirical results not sensitive to minor changes in either this percentage or in the order of the projection model used.

- 2.3.1 "Full Disaggregation" Results
- 2.3.2 "Polynomial Smoothed" Results
- 2.3.2 "Three-Band Model" Results

- Section 2.4 'New Economy' Versus 'Old Economy' Comparison
- Section 2.5 Comparison to A Policy Rule Estimated Using A Conventional (Moving Average) Natural Rate of Unemployment Decomposition

In Section 2.2 the usual Taylor-type monetary policy rule model discussed as Equation (1) in Section 1.1 is generalized to allow for a time-varying natural real interest rate and for dynamics, in the form of lags in i_t. The latter generalization is econometrically necessary, so as to yield model errors free from serial correlation; and it is economically interesting, in that such dynamics correspond to the FOMC acting so as to smooth the time path of the federal funds rate. In Section 2.3 the model is extended to allow for frequency dependence in the coefficients on π_t and u_t – as in Equation (10) of Section 4.1. In Sections 2.3 through 2.5 we obtain and discuss the central empirical results of the paper.

Finally, before turning to our regression model estimates, it is useful to consider time plots of the frequency component time series of π_t and u_t, obtained using the one-sided real-time decomposition analysis described above and detailed in Section 4.1 of the Technical Appendix. The time plots of these components should be smoother for the lower frequency components, and more noisy-looking for the higher frequency components.

As shown in Section 4.1, a window of length 36 months allows for 19 distinct frequency components; analogously, a window of length 120 months allows for 61 distinct frequency components.[16] Thus, displaying these time plots for all of the possible frequency component time series of both π_t and u_t (with a 36-month and 120-month long windows, respectively) would require nearly 100 plots. Consequently, we instead present these plots here for what we call

[16] Because these window lengths are even (rather than odd) numbers, the number of allowed non-zero frequencies is one half of the window length. We also obtain a "zero-frequency" component which is obtained from estimating a linear time trend regression for the data in each window. This zero-frequency component is thus extracting an adaptive, backward-looking nonlinear trend time series for each of π_t and u_t, estimated using the sample data in each window. Note that this procedure is not equivalent to a one-sided moving-average trend estimate and hence avoids the poor properties inherent in such moving-average estimates, such as distorted turning points. Section 4.1 provides more detail with respect to these issues.

the "calendar-based 3-band" decomposition below, which aggregates these 19 or 61 frequency components in an economically reasonable way:

- "Low-Band" corresponding to reversion periods greater than 36 months
- "Mid-Band" corresponding to reversion periods between 12 and 36 months, inclusive
- "High-Band" corresponding to reversion periods less than 12 months

Table 1 displays the 19 frequencies (and corresponding reversion periods) allowed by a 36-month window, for example. These frequencies are equi-spaced. But reference to Table 1 shows that (because they are proportional to the reciprocals of the frequencies) the corresponding reversion periods are decidedly not equi-spaced. Consequently, these three bands contain (i.e., aggregate together) unequal fractions of the 19 distinct frequency components. In particular, the low-band contains only the zero-frequency component, which – as noted above – is basically a backward-looking nonlinear trend estimate, extracted using linear time trend regression within each window. The 'mid-band' at this window length aggregates together three frequency components, with reversion periods of 12, 18, and 36 months, respectively; and the 'high-band' aggregates the remaining fifteen distinct frequency components, with reversion periods ranging from 2 to 9 months. The time plots for these three components, partitioning π_t and u_t and displayed as Figures 1 and 2, behave just as one might expect: the low-band plots are quite smooth, the high-band plots look very noisy, and the mid-band plots are intermediate in smoothness.

2.2 Results for An Ordinary (But Dynamic) Taylor-type Policy Rule Model Ignoring Possible Frequency Dependence

Our empirical specification was guided by theoretical considerations, in conjunction with a desire to allow the data to speak as clearly as possible. First, the federal funds rate is highly persistent. Empirical estimates of central bank policy functions from around the world generally indicate the presence of substantial inertia (see, e.g., Goodhart 1999, Coibion and Gorodnichenko 2012), consistent with partial adjustment of the interest rate toward its target. Incorporating such inertia is also found to improve outcomes in many theoretical models (see, e.g., Woodford 2003, Taylor and Williams 2011). In the U.S. at least, there is evidence for double inertia (see

Carlstrom and Fuerst, 2014): two lags of the federal funds rate are necessary to yield serially uncorrelated fitting errors.

Second, we do not wish to impose a fixed equilibrium or natural real interest rate over time. Laubach and Williams (2003, 2015) estimate the natural rate of interest and find that those estimates have varied significantly through U.S. history. Our model specification therefore includes a time-varying natural interest rate (r_t^*), and we use the previous-quarter Laubach-Williams estimate for this entity. The Laubach-Williams methodology is the most well-known method for estimating this variable; estimates are now updated quarterly and made publicly available on the website of the Federal Reserve Bank of San Francisco. Unfortunately, such estimates – regardless of method – "are very imprecise and subject to considerable real time mismeasurement" (LW 2003). To the extent that these real time estimates are inaccurate, our coefficient estimates will be distorted. Still, we judge that including noisy estimates is better than implicitly assuming that the natural rate of interest is fixed.

Third, our method gracefully allows the data to speak about how the central bank appears to approximate, and respond to, time-varying entities such as the natural rate of unemployment, or about its behavior vis-à-vis different frequency components of inflation; and inflation; and we include a constant term to pick up a fixed inflation target, if that exists. Accordingly, except for using the Laubach and Williams estimate of the natural rate of interest, we do not include any such time-varying entities. Thus, our specification is given by:

$$i_t = \delta_1 i_{t-1} + \delta_2 i_{t-2} + (1 - \delta_1 - \delta_2)(\alpha + r_t^* + \phi_\pi \pi_t + \phi_u u_t) + e_t. \tag{2}$$

As is standard in the literature, lagged values of π_t and u_t are not included. Because of the parameter δ, Equation (2) must be estimated via non-linear least squares (NLS) instead of OLS. The term $(\alpha + r_t^* + \phi_\pi \pi_t + \phi_u u_t)$ is often interpreted as a target interest rate, with the central bank eliminating a fraction $(1 - \delta_1 - \delta_2)$ of the gap between the current target and the current federal funds rate each month.[17]

[17] Several of the terms in Equation (2) are extremely persistent. While the presence of lagged interest rate terms ensures that all of the coefficients will be estimated consistently, Ashley and Verbrugge (2009a) have shown that a degree of finite-sample bias can be expected with persistent regressors. Our sample lengths here – of $T = 176$ in the MBM period and $T = 348$ in the VGB period – are not small, but we note that their simulation results warn that these biases are not necessarily completely negligible when there is substantial regressor persistence, even with several hundred observations. It is worth pointing out that unit root hypotheses are all strongly rejected

Table 3 displays NLS estimates of δ_1, δ_2, ϕ_π, and ϕ_u separately over the MBM and VGB subperiods defined in Section 2.1. As noted above, the inclusion of two lags in i_t in this model (and in the models discussed in Sections 2.3 and 2.4 below, as well) suffices to yield serially uncorrelated model fitting errors; Eicker-White standard errors are quoted for all coefficient estimates, here and below, to account, at least asymptotically, for any heteroskedasticity in ϵ_t.[18] The coefficients δ_1 and δ_2 can be taken to quantify 'interest rate smoothing' behavior by the FOMC, so it is noteworthy that the null hypothesis that both of these coefficients are zero can be rejected with $P < 0.0005$ in these two models and, indeed, in all of the models estimated here.

At this point ignoring any possible dependence of ϕ_π and ϕ_u on the persistence of the fluctuations in the observed values of π_t and u_t, the coefficient estimates in Table 3 indicate that the FOMC exhibited statistically significant policy responses to fluctuations in π in both the MBM and VGB periods. The FOMC's response to inflation rate fluctuations was notably smaller in the MBM than in the VGB period, however: on average the FOMC increased the federal funds rate by only 0.70% for every 1% increase in the inflation rate in MGM period, whereas in the VGB period the estimated response is 1.4%. In contrast, the FOMC's response to a 1% increase in the unemployment rate is statistically significant only in the MBM period, and of only modest economic significance (-0.91%) even then.[19]

Section 2.3 below presents evidence that these results are actually artifactual: allowing for frequency dependence in the two policy reaction coefficients yields interestingly different results, indicating that the omission of distinct frequency components of π_t and u_t in Equation (2) is

in each of the MBM and VGB subperiods when these periods are considered separately. This is in keeping with what one might expect, given the arguments in Clarida, Gali and Gertler (1999) to the effect that stationarity for these variables is implied by theoretical models in which Taylor-type monetary policies play a role. Partitioning the sample also, to some degree, alleviates the problem of time-varying variance mentioned in Sims and Zha (2001, 2006), but we use Eicker-White standard error estimates throughout nonetheless.

[18]Possible parameter estimation distortion due to three outlying observations in the fitting errors – for July 1973, May 1980, and Feb 1981 – was addressed using dummy variables to shift the intercept. The estimated coefficients on these dummy variables were always highly significant – and (negative, negative, positive) in signs, respectively – but their exclusion did not substantively affect the inference results reported below. Consequently, the listing of these coefficient estimates – and the model intercept term (α) – is, for simplicity, suppressed in the results tables here.

[19]Section 2.5 re-estimates Equation (2) using a conventional unemployment rate gap, and finds – in keeping with most of the literature – evidence for a response to unemployment rate fluctuations in the VGB period. As is evident from those results, the apparent non-response to unemployment rate fluctuations in the VGB period is actually arising from unmodeled frequency dependence in the relationship between the federal funds rate and the unemployment rate.

so substantially mis-specifying this Taylor-type monetary policy as to yield seriously misleading conclusions as to the FOMC's past behavior.

2.3 Results for the Frequency Dependent Model

Here we re-specify Equation (2) to allow for the possibility that the coefficients on π_t and u_t depend on the persistence levels of the fluctuations in these variates. This yields the model:

$$i_t = \delta_1 i_{t-1} + \delta_2 i_{t-2} + (1 - \delta_1 - \delta_2)\left(\alpha + r_t^* + \sum_{j=1}^{10} \phi_{\pi,j}\pi_t^j + \sum_{k=1}^{10} \phi_{u,k} u_t^k\right) + \epsilon_t. \tag{3}$$

As detailed in the Technical Appendix (Section 4.1), the real-time inflation rate and real-time unemployment rate series are each decomposed into ten frequency component series. Thus, π_t^1 is the lowest frequency component of inflation, and $\phi_{\pi,1}$ is the coefficient on this component; and so on.[20]

2.3.1 "Full Disaggregation" Results

In the columns headed "Full Disaggregation" Table 4 presents our NLS estimation results for the inflation-rate policy response coefficients $(\phi_{\pi,1}, ..., \phi_{\pi,10})$ and unemployment-rate policy response coefficients $(\phi_{u,1}, ..., \phi_{u,10})$ in Equation (3) over the two sample sub-periods, MBM and VGB.

Identifying the lowest-frequency component of the unemployment rate – i.e., (u_t^1) – in this model as the 'natural rate of unemployment,' and supposing, as is commonly done, that the FOMC responds weakly or not at all to perceived fluctuations in this natural rate, one would expect that $\phi_{u,1}$ would be negligible. This expectation is satisfied for both the MBM period coefficient (0.358 ± 0.200) and for the VGB period coefficient (-0.070 ± 0.441).

The rows at the foot of Table 4 display the p-values at which two null hypotheses with regard to the inflation response coefficients $(\phi_{\pi,1}, ..., \phi_{\pi,10})$ and the unemployment rate response coefficients $(\phi_{u,1}, ..., \phi_{u,10})$ can be rejected.

[20]Recall that the nine highest-frequency components of π_t and the 51 highest-frequency components of u_t, with reversion periods less than or equal to four months, are aggregated together. Thus, there are only ten components for each of these variates in Equation (3) rather than the nineteen components possible with a window of length thirty six months (for π_t) and the 61 components possible with a window length of 120 months (for u_t). See Table 1 (and the latter portion of Section 4.1) for details.

The first set of tests address the issue of whether the FOMC's policy reaction function pays any attention at all to fluctuations the inflation and unemployment rates: if the coefficients on $\phi_{\pi,1}, ..., \phi_{\pi,10}$ are all zero, then there is no dependence in the FOMC's reactions to fluctuations in π_t. Similarly, if the coefficients on $\phi_{u,1}, ..., \phi_{u,10}$ are all zero, then there is no dependence in the FOMC's reactions to fluctuations in u_t. Note that there is very strong evidence that the FOMC pays attention to fluctuations in both the inflation rate and in the unemployment rate in both periods. In contrast, Equation (2) – the (linear) model not allowing for frequency dependence – erroneously suggests that the FOMC largely ignored unemployment rate fluctuations in the MBM period.

The second set of tests addresses the issue of frequency dependence: if the coefficients on $\phi_{\pi,1}, ..., \phi_{\pi,10}$ are all equal, then there is no frequency (or persistence) dependence in the FOMC's reactions to fluctuations in π_t. Similarly, if the coefficients on $\phi_{u,1}, ..., \phi_{u,10}$ are all equal, then there is no frequency (or persistence) dependence in the FOMC's reactions to fluctuations in u_t. Note that there is no evidence for frequency dependence in the FOMC's reactions to fluctuations in π_t in the MBM period, whereas there is very strong evidence for frequency dependence in the unemployment rate response coefficient using the data for this period. And, in the VGB period, there is very strong evidence for frequency dependence in the FOMC's reactions to fluctuations in both variables – notably, in this period the FOMC's response to inflation fluctuations appears to change sign and become inverse for fluctuations with a reversal period of 9 months.

The interpretation of the individual estimated response coefficients for particular frequency bands – e.g. $\hat{\phi}_{u,9} = -11.405 \pm 4.780$ for the model estimated using data from the MBM period – is not particularly useful. This is because actual fluctuations in the inflation rate themselves contain an array of persistence levels: one is never going to see an inflation rate fluctuation which is a pure sinusoid with a period of exactly 9 months, for example. Still, one expects to find that the individual $\hat{\phi}_{\pi,1}, ..., \hat{\phi}_{\pi,10}$ coefficients are mostly – albeit not entirely – either positive or statistically insignificant and this is so for both the MBM and VGB periods. Similarly, one expects to find that the individual $\hat{\phi}_{u,1}, ..., \hat{\phi}_{u,10}$ coefficients are mostly either negative or statistically insignificant and this, too, is so for both the MBM and VGB periods.

The testing results discussed above clearly indicate that there is a non-constant pattern – i.e. frequency or persistence dependence – in the FOMC's reaction coefficients. The characterization and interpretation of these patterns is hindered by the substantial amount of sampling variation in the individual coefficient estimates, however.

2.3.2 "Polynomial Smoothed" Results

The substantial sampling variation in the $\hat{\phi}_{\pi,1}, ..., \hat{\phi}_{\pi,10}$ and $\hat{\phi}_{u,1}, ..., \hat{\phi}_{u,10}$ coefficient estimates motivated us to sharpen these estimates by assuming that they vary smoothly across the 10 frequency bands. This smoothing was implemented by assuming that the frequency variation in each of these parameter groups can be captured by an m^{th} order polynomial and instead estimating the $(m+1)$ coefficients in each polynomial. This tactic both smooths the parameter variation across the frequency bands and – presuming that $m \ll 10$ – also substantially reduces the number of parameters to be estimated. Here we found that the coefficient corresponding to $m = 3$ was statistically insignificant in all four cases, so quadratic polynomial smoothing was used.[21] These estimates are displayed in the two "Polynomial Smoothed" columns of Table 4.

The patterns of the inflation rate and unemployment rate reaction coefficients now obtained are similar to those for the "full disaggregation" model estimations, but appear to be more precisely determined – especially so for the $\hat{\phi}_{u,j}$ estimates. From these results we again – now more reliably – conclude that the FOMC responded in both periods negatively to unemployment shocks and positively to low-frequency (relatively persistent) inflation shocks, as one might expect. Again there is strong statistical evidence for frequency dependence with respect to the FOMC's reactions to fluctuations in these variables. Indeed it is this frequency dependence which notably distinguishes the nature of the FOMC's responses to the real-time observations on these two variables: For one thing, we find clear evidence of frequency dependence in the FOMC's inflation rate fluctuation responses only in the VGB period – not in the the MBM period. And, while there is also strong evidence that the FOMC did respond to fluctuations in the unemployment rate in both periods, we find clear evidence that the FOMC responded directly to short-term – i.e., higher frequency or less persistent – fluctuations in the inflation rate in the MBM period, whereas – in contrast – we see that the FOMC's response to inflation rate fluctuations actually becomes inverse in VGB period for inflation fluctuations with a reversion

[21]This kind of smoothing was introduced by Almon (1965); see Johnston (1972, pp. 294-295) for a condensed description.

period of 18 months or less.

2.3.3 "Three-Band Model" Results

In this section we analyze the estimated frequency response coefficients ($\hat{\phi}_{\pi,1}, ..., \hat{\phi}_{\pi,10}$ and $\hat{\phi}_{u,1}, ..., \hat{\phi}_{u,10}$) aggregated and smoothed in a different way. This approach is more 'calendar driven' than the polynomial-smoothing approach – and thus, perhaps, a bit *ad hoc* – but it likely corresponds fairly closely to the way the FOMC itself views the real-time data with which it conducts actual monetary policy. In particular, we aggregate the full number of mathematically distinct frequency components implied by our moving windows into just three bands:

- "Low Frequency Band (reversion periods: > 36 months)
- "Medium Frequency Band (reversion periods: ≥ 12 months and ≤ 36 months)
- "High Frequency Band (reversion periods: < 12 months)

The Low Frequency band comprises what we think the FOMC takes to be 'long-term' or even 'permanent' fluctuations. The Medium Frequency band – consisting of the three components with reversion periods of 12, 18, and 36 months – comprise the fluctuations which we think the FOMC considers 'business cycle variations.' And the High Frequency band – consisting of all the components with reversion periods ranging from 2 months up to 9 months – comprise what we think the FOMC considers to be a combination of current news ('shocks') and measurement noise.

Returning to Table 4, note that the this aggregation did not noticeably impact the adjusted R^2 for the regression model, so the parameter restrictions imposed by this aggregation are not materially affecting the model's ability to model the i_t variation in either period. Consistent with this observation, the pattern of the inference results – both with regard to testing the null hypotheses that the response coefficients are zero and with regard to testing whether they vary across the frequency bands – are all quite similar to those reported for the 'polynomial smoothed' approach. In particular, there is strong evidence that observed fluctuations in the inflation rate affect the FOMC's behaviour – and in a significantly frequency dependent manner – in both the MGM and VGB periods. The same is true for observed fluctuations in the unemployment rate, but only in the VGB period.

Turning to the coefficient estimates for the individual bands, the Low Frequency band coefficient on unemployment rate fluctuations is statistically insignificant for both periods, supporting the notion that the FOMC considers these to primarily be changes in the natural rate, whereas the statistical evidence for the FOMC responding to unemployment rate fluctuations in the Medium and High frequency bands is strong – and with a sensible (negative) sign in both periods, but much stronger in the VGB period.

With regard to the inflation rate reaction coefficients, in the MGM period the FOMC strongly – and in the expected, direct manner – to Low Frequency band (high persistence) fluctuations in both periods, but does not respond to Medium Frequency band inflation rate fluctuations in either period. In contrast, the FOMC's response to High Frequency band inflation rate fluctuations (with reversion periods less than a year) is significantly inverse – both statistically and economically – in the VGB period.

2.4 Comparison with the Ball and Tchaidze (2002) Results

As we noted at the end of Section 1.1, Ball and Tchaidze (2002) rely upon a time-varying NAIRU estimate to explain the apparent change in FOMC behavior during the new economy period. Upon the inclusion of this estimate, FOMC behavior no longer appears different during this period.

In this Section, we revisit this issue. Our specification departs from this earlier study in several ways. First, following now standard practice in the literature, our estimated monetary policy rule includes an estimate of the natural real interest rate and interest rate smoothing. Uniquely, it also allows for a different response to fluctuations of different frequencies. Because of this, the specification gracefully includes an (implicit) natural rate of unemployment estimate, namely the lowest-frequency fluctuations in the unemployment rate. While this natural rate estimate is thus quite different from most other studies, it shares the characteristic with the estimate used in Ball and Tchaidze (2002) that it is a real-time estimate that policymakers could have observed.[22] Ball and Tchaidze (2002) date the new economy period as starting in 1996:1, to correspond to an acceleration in productivity growth estimated in Ball and Moffitt

[22]See Section 2.5 for results comparing the use of this natural rate of unemployment estimate to a one-sided moving average estimate, per recent suggestions.

(2001), since they note that many authors suggest that the FOMC deviated from its normal behavior because it recognized changes in the economy, such as higher productivity growth. Using this reasoning, we instead date the 'new economy' period as starting one year earlier, in 1995:1, to correspond to the date when productivity growth switched from negative to positive, according to estimates made by the Federal Reserve Bank of San Francisco using the methods of Basu, Fernald, and Kimball (2006). Since Ball and Tchaidze date the old economy period to start when Greenspan first became chair, i.e., the fourth quarter of 1987, our choice makes the two periods more balanced, without changing the tenor of the original findings.

Table 5 presents monetary policy rule coefficient estimates that correspond to the baseline estimates of Ball and Tchaidze (2002), with no decomposition by frequency, though using a monetary policy rule in the form of Equation (2). Our estimates are somewhat unusual in that we do not find convincing evidence that the FOMC responds to inflation in either period. This results in part from our use of headline CPI inflation rather than core CPI inflation, and in part from specifying a richer policy rule that must be estimated using nonlinear methods. (But with a simple linear specification, we find statistical significance in both periods, even using headline CPI inflation.) Still, in keeping with the original article, our Table 5 results would support the conclusion that the FOMC appears to have greatly reduced its response to unemployment rate fluctuations during the new economy period.

Table 6 presents the corresponding results after disaggregating fluctuations in both independent variables into the three frequency bands used in the previous section. We still do not find evidence for an FOMC response to inflation rate fluctuations in the 'old economy' period. We do, however, reach the same conclusions as did Ball and Tchaidze (2002): upon appropriately allowing for natural rate variation in the policy rule – in our case through decomposition of the explanatory variables by frequency – it becomes evident that the FOMC did in fact respond to unemployment rate fluctuations during the 'new economy' period.[23]

Our results are actually more striking than those of Ball and Tchaidze (2002): The conventional wisdom is that the early 1990s were ordinary, but that "Greenspan's Fed" supposedly

[23]We interpret the negative coefficient estimate on medium-frequency fluctuations in the inflation rate during the 'new economy' period as sampling variation in our nonlinear least squares estimates, due to an isolated year-long episode during which the inflation rate was falling and yet the FOMC left the federal funds rate unchanged.

'deviated from its normal behavior' by holding interest rates steady despite a booming economy and falling unemployment that would normally have triggered a tightening. (Ball and Tch. p.108) during the 'new economy' period. Our results reinforce Ball and Tchaidze's finding that the monetary authority certainly did respond to unemployment rate changes in the latter period; and our results are also fully consistent with their conclusion a 'fixed natural rate' assumption leads to misleading conclusions. Strikingly, however, we actually find more evidence than they do of a response to inflation in the 'new economy' period.

2.5 Comparison to A Policy Rule Estimated Using A Conventional (Moving Average) Natural Rate of Unemployment Decomposition

In this section, we compare the relationship of the federal funds rate to our implicit estimate of the natural rate of unemployment – based on the lowest frequency unemployment rate component – versus its relationship to another prominent natural rate estimate, a one-sided moving average of the type used in Boivin (2006) or Bernanke and Boivin (2003), using regression models estimated over the VGB period (September 1979 to August 2008).[24]

Here we estimate a more parsimonious model than above, based on a specification that more closely parallels traditional specifications, which merely separate the unemployment rate into two parts (i.e., estimate an unemployment rate gap), and which do not separate the inflation rate into component parts at all. In particular, we reduce the number of coefficient estimates to two for the unemployment rate, and to two for the inflation rate.[25] More specifically, the model specifications now include u_t^*, $u_t - u_t^*$, π_t^*, and $\pi_t - \pi_t^*$, where π_t^* refers to our lowest frequency component of the inflation rate, and u_t^* is alternatively either a one-sided nine-year moving average natural rate estimate or is the lowest-frequency component of the unemployment rate. The resulting regression model estimation results are summarized in Table 7.

[24] Boivin (2006) and Bernanke and Boivin (2003) make use of a one-sided five year moving average. However, we use a one-sided nine-year moving average because, as pointed out by a referee, this choice results in the greatest coherence with a simple average of three real-time natural rate estimates: from the Greenbook, the Congressional Budget Office, and the Survey of Professional Forecasters.

[25] Does this "two-band" model suffice over the VGB period? We find that the null hypothesis that $\phi_{u,2} = ... = \phi_{u,10}$ is easily rejected by the data using either the full disaggregation or three-band model specification, while the analogous null hypothesis that $\phi_{\pi,2} = ... = \phi_{\pi,10}$ yields mixed results across the two models.

We view these model estimates as supportive of our earlier results: Our frequency component based natural rate of unemployment estimate is unrelated to federal funds rate movements, consistent with theory, whereas the moving-average natural rate estimate is, in contrast, positively and significantly related to the federal funds rate. Moreover, the model using our natural rate of unemployment estimate yields a coefficient estimate on the unemployment rate gap which is larger in magnitude, albeit less precisely determined. Both of these results suggest that our frequency-decomposition-based estimate is a better proxy for the underlying natural rate of unemployment.

3 Conclusions

This paper presents a practical implementation of a new way of specifying an econometric regression model, allowing for flexible disaggregation of one or more of the explanatory variables – which may be real-time measures and/or in feedback with the dependent variable – into frequency (or persistence) components which add up to the original sample data. This decomposition allows us obtain richer conclusions as to how fluctuations in these explanatory variables with a distinct level of persistence will impact the dependent variable.

This new estimation technology is here applied to the estimation and analysis of Taylor-type monetary response policy functions that are increasingly used to model the behavior of the U.S. central bank, first in the Martin-Burns-Miller (MBM) period and then in the Volcker-Greenspan-Bernanke (VGB) period. We find strong evidence for frequency dependence in the FOMC's policy responses to both inflation and unemployment rate fluctuations in the VGB period, but we find evidence for frequency dependence only in its responses to unemployment rate fluctuations in the MBM period – indeed, it is the differing pattern of frequency dependence in the FOMC's reaction coefficients which most clearly distinguishes the FOMC's behavior in these two periods. Interestingly, in the VGB period we find strong evidence that the FOMC's response to inflation rate shocks actually becomes significantly *inverse* – both statistically and economically – for shocks which reverse within a year. *Notably, the estimation results ignoring the frequency dependence in the FOMC's monetary responses give no hint of this inverse response to inflation rate shocks in the VGB period. This finding is an example of the sort of unexpected* *empirical discovery which our new method can uncover.*

Thus, we reach three general conclusions in this paper:

1) Frequency dependence is statistically and economically important in analyzing the FOMC's historical monetary policy rules. Policy has clearly distinguished between (and reacted differentially to) inflation and unemployment rate fluctuations of varying degrees of persistence; and the pattern of these differential reactions is interestingly specific to which chairmanship period is addressed.

2) The empirical analysis of historical monetary policy reaction rules and – by extension – other macroeconomic relationships, without giving the data a chance to appropriately allow for frequency/persistence dependence in at least some key coefficients, is quite likely to miss out on uncovering some interesting features in the data. Distorted, and unnecessarily over-simplified inferences are thus likely consequences of ignoring frequency dependence in estimating such relationships.

3) Economic theory often suggests that decision-makers distinguish between fluctuations with different degrees of persistence, or that the relationship between two variables is likely to differ by frequency. The relatively straightforward technique put forward here illustrates how data on a macroeconomically important relationship can be disaggregated by frequency – even in settings where feedback is likely, so that ordinary spectral analysis is inappropriate – so that an econometric analysis can allow for (and test for) such frequency dependence. We believe that this new empirical tool can lead to sharp new insights about the process generating such data, whereas ordinary time-series techniques (in either the time domain or the frequency domain) would fail to reliably uncover features of this nature in the data generating process.

References

[1] **Almon, Shirley** (1965): "The Distributed Lag between Capital Appropriations and Expenditures," *Econometrica*, 30, 178-196.

[2] **Ashley, Richard and Randal J. Verbrugge** (2007a): "Mis-Specification and Frequency Dependence in a New Keynesian Phillips Curve," working paper.

[3] **Ashley, Richard and Randal J. Verbrugge** (2007b): "Mis-Specification in Phillips Curve Regressions: Quantifying Frequency Dependence in This Relationship While Allowing for Feedback," working paper.

[4] **Ashley, Richard and Randal J. Verbrugge** (2009a): "To Difference or Not to Difference: A Monte Carlo Investigation of Inference in Vector Autoregression Models," *International Journal of Data Analysis Techniques and Strategies* Vol. 1, No. 3, 242-274.

[5] **Ashley, Richard and Randal J. Verbrugge** (2009b): "Frequency Dependence in Regression Model Coefficients: An Alternative Approach for Modeling Nonlinear Dynamic Relationships in Time Series,"*Econometric Reviews*, 28(1-3), 4-20.

[6] **Ashley, Richard and Kwok Ping Tsang** (2013): "The Oil Price-Macroeconomy Relationship: Does Persistence Matter? " working paper.

[7] **Ashley, Richard and Guo Li** (2014): "Re-Examining the Impact of Housing Wealth and Stock Wealth on Retail Sales: Does Persistence in Wealth Changes Matter?," *Journal of Housing Economics*, 26, 109-118.

[8] **Ball, Laurence and Robert R. Tchaidze** (2002): "The Fed and the New Economy", *American Economic Review*, Vol. 92, No. 2, Papers and Proceedings of the One Hundred Fourteenth Annual Meeting of the American Economic Association, pp. 108-114.

[9] **Basu, Susanto, Fernald, John G. and Miles S. Kimball** (2006) "Are Technology Improvements Contractionary?," *American Economic Review*, Vol. 96, No. 5, 1418-1448.

[10] **Bernanke, Ben S. and Jean Boivin** (2003) "Monetary policy in a data-rich environment," *Journal of Monetary Economics*, vol. 50, no. 3, p. 525-546.

[11] **Boivin, Jean** (2006) "Has U.S. Monetary Policy Changed? Evidence from Drifting Coefficients and Real-Time Data," *Journal of Money, Credit and Banking*, vol. 38, no. 5, p. 1149-1173.

[12] **Box, George E.P. and Gwilym Jenkins** (1976): *Time Series Analysis, Forecasting and Control*, Holden-Day, San Francisco.

[13] **Brischetto, Andrea, and Anthony Richards** (2007), "The Performance of Trimmed Mean Measures of Underlying Inflation," paper presented at the Conference on Price Measurement for Monetary Policy, May 24-25, 2007, Jointly sponsored by the Federal Reserve Banks of Cleveland and Dallas.

[14] **Bryan, Michael F. and Brent Meyer** (2002): "Are Some Prices in the CPI More Forward Looking Than Others? We Think So", *Economic Commentry*, No. 2010-2, Federal Reserve Bank of Cleveland.

[15] **Carlstrom, Charles T. and Timothy S. Fuerst** (2014): "Adding Double Inertia to Taylor Rules to Improve Accuracy" *Federal Reserve Bank of Cleveland Economic Commentary 2014-08*.

[16] **Carlstrom, Charles T. and Timothy Stehulak** (2015): "The Long-Run Natural Rate of Interest" *Federal Reserve Bank of Cleveland Economic Trends*.

[17] **Cecchetti, Stephen G.** (1997): "Measuring Short-Run Inflation For Central Bankers", *FRB Saint Louis - Review*, , vol. 79, no. 3, p. 143-155.

[18] **Christiano, Lawrence J. and Terry J. Fitzgerald**(2003): "The Band Pass Filter," *International Economic Review*, vol. 44, no. 2, p. 435-465.

[19] **Clarida, Richard, Jordi Gali and Mark Gertler** (2000): "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics*, vol. 115, no. 1, p. 147-180.

[20] **Cochrane, John H.** (1989): "The Return of the Liquidity Effect: A Study of the Short-Run Relation between Money Growth and Interest Rates", *Journal of Business and Economic Statistics*, Volume 7, 75-83.

[21] **Cogley, Timothy, Riccardo Colacito and Thomas J. Sargent** (2007): "Benefits from U.S. Monetary Policy Experimentation in the Days of Samuelson and Solow and Lucas," *Journal of Money, Credit and Banking*, Vol.39, No.1, 67-99.

[22] **Coibion, Olivier, and Yuriy Gorodnichenko** 2012: "Why Are Target Interest Rate Changes So Persistent?" *American Economic Journal: Macroeconomics*, 4(4): 126-62.

[23] **Consolo, Agostino and Carlo A. Favero** (2009): "Monetary Policy Inertia: More a Fiction than a Fact?" *Journal of Monetary Economics*, Volume 56, Issue 6, 900-906.

[24] **Curdia, Vasco, Andrea Ferrero, Ging Cee Ng, and Andrea Tambalotti** (2015): "Has U.S. Monetary Policy Tracked the Efficient Interest Rate?" *Journal of Monetary Economics*, 70, 72-83.

[25] **Dagum, Estela** (1978): "Modelling, Forecasting and Seasonally Adjusting Economic Time Series with the X11 ARIMA Method," *The Statistician*, 27, 3/4, 203-216.

[26] **Detmeister, Alan, and Nathan Babb** (2017): "Nonlinearities in the Phillips Curve for the United States," Federal Reserve Bank of Cleveland, Working Paper.

[27] **Dolmas, Jim and Mark A.Wynne** (2008): "Measuring Core Inflation: Notes from a 2007 Dallas Fed Conference," Staff Papers No. 4, May 2008, Federal Reserve Bank of Dallas.

[28] **Engle, Robert** (1974): "Band Spectrum Regression," *International Economic Review*, 15, 1-11.

[29] **Friedman, Milton** (1968): "The Role of Monetary Policy". *American Economic Review* 58: 1-17.

[30] **Gali, Jordi** (2009): *Monetary Policy, Inflation, and the Business Cycle*, Princeton University Press.

[31] **Giannone, Domenico, and Troy D. Matheson** (2007): "A New Core Inflation Indicator for New Zealand," *International Journal of Central Banking*, December, p. 145-180.

[32] **Goodhart, Charles A.** (1999): "Central Bankers and Uncertainty" Keynes Lecture in Economics *Proceedings of the British Academy*, 101: 229-71.

[33] **Gordon, Robert J.** (1975): "Alternative Responses of Policy to External Supply Shocks," *Brookings Papers on Economic Activity*, 1975(1), p. 183-206.

[34] **Granger, Clive W.J.** (1969): "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods," *Econometrica*, 37, 424-438.

[35] **Hamilton, James D., Ethan S. Harris, Jan Hatzius, and Kenneth D. West.** (2015): "The Equilibrium Real Funds Rate: Past, Present, and Future" (Presented at the US Monetary Policy Forum, New York, February 27, 2015.)

[36] **Hannan, E.** (1963): "Regression for Time Series," in M. Rosenblatt, ed. *Time Series Analysis*, John Wiley: New York, 14.37.

[37] **Higgins, Amy, and Randal Verbrugge** (2015): "Tracking Trend Inflation: Nonseasonally Adjusted Variants of the Median and Trimmed-Mean CPI," Federal Reserve Bank of Cleveland, Working Paper p. 15-27.

[38] **Holston, Kathryn, Thomas Laubach, John C. Williams.** (2016): "Measuring the Natural Rate of Interest: International Trends and Determinants." Federal Reserve Bank of San Francisco Working Paper 2016-11. http://www.frbsf.org/economic-research/publications/workingpapers/ wp2016-11.pdf

[39] **Johnston, J.** (1972) *Econometric Methods*, McGraw-Hill: New York.

[40] **Jondeau, Eric, Herve Le Bihan and Clementine Galles** (2004): "Assessing Generalized Method-of-Moments Estimates of the Federal Reserve Reaction Function," *Journal of Business and Economics Statistics*, 22(2), 225-239.

[41] **Kim, Chang-Jin and Charles R. Nelson** (2006): "Estimation of a Forward-Looking Monetary Policy Rule: A Time-Varying Parameter Model Using Ex Post Data," *Journal of Monetary Economics*, 53, p. 1949-1966.

[42] **Knotek, Edward S. II, Randal Verbrugge, Christian Garciga, Caitlin Treanor, and Saeed Zaman** (2016): "Federal Funds Rates Based on Seven Simple Monetary Policy Rules." *Federal Reserve Bank of Cleveland Economic Commentary 2016-17.*

[43] **Koopmans, Lambert H.** (1974): *The Spectral Analysis of Time Series*, San Diego, CA: Academic Press.

[44] **Laubach, Thomas, and John C. Williams.** (2003): "Measuring the Natural Rate of Interest" *Review of Economics and Statistics* 85(4), 1063-1070.

[45] **Laubach, Thomas, and John C. Williams.** (2015): "Measuring the Natural Rate of Interest Redux" Federal Reserve Bank of San Francisco Working Papers, 2015-16.

[46] **McConnell, Margaret M. and Gabriel Perez-Quiros** (2000): "Output Fluctuations in the United States: What Has Changed Since the Early 1980's?" *American Economic Review*, Vol. 90, No. 5, pp. 1464-1476.

[47] **McCulloch, J. Huston** (2007): "Adaptive Least Squares Estimation of the Time-Varying Taylor Rule," working paper.

[48] **Meltzer, Allan H.** (2005): "Origins of the Great Inflation," *Federal Reserve Bank of St. Louis Review*, March/April Part 2, p. 145-176.

[49] **Meyer, Brent, Guhan Venkatu and Saeed Zaman** (2013): "Forecasting Inflation? Target the Middle," *Economic Commentary*, 2013-05, pp. 1-4.

[50] **Mishkin, Frederic S.** (2007): "Inflation Dynamics" in *International Finance*, vol. 10(3), Blackwell Publishing: Oxford, p. 317-334.

[51] **Newey, Whitney K. and Kenneth D. West** (1987): "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, Vol. 55, No. 3, p. 703-708.

[52] **Orphanides, Athanasios** (2001): "Monetary Policy Rules Based on Real-Time Data," *American Economic Review*, vol. 91, no. 4, p. 964-985.

[53] **Orphanides, Athanasios** (2002): "Monetary-Policy Rules and the Great Inflation," *American Economic Review: Papers and Proceedings*, vol. 92, no. 2, p. 115-120.

[54] **Orphanides, Athanasios** (2008): "Taylor Rules,"in *The New Palgrave Dictionary of Economics*. Second Edition. Steven N. Durlauf and Lawrence E. Blume (eds.) Palgrave Macmillan, 2017. *The New Palgrave Dictionary of Economics Online*. Palgrave Macmillan. (doi: 10.1057/9780230226203.1686)

[55] **Phelps, Edmund S.** (1968). "Money-Wage Dynamics and Labor-Market Equilibrium". *Journal of Political Economy* 76: 678–711.

[56] **Rudebusch, Glenn D.** (2002): "Term Structure Evidence on Interest Rate Smoothing and Monetary Policy Inertia," *Journal of Monetary Economics*, Volume 49, Issue 6, 1161-1187.

[57] **Sims, Christopher A. and Tao Zha** (2001): "Stability and Instability in US Monetary Policy Behavior," working paper.

[58] **Sims, Christopher A. and Tao Zha** (2006): "Were There Regime Switches in U.S. Monetary Policy?" *American Economic Review*, vol. 96, no. 1, p. 54-81.

[59] **Stock, James and Mark Watson** (1999): "Business Cycle Fluctuations in U.S. Macroeconomic Time Series," in (John Taylor and Michael Woodford, eds) *Handbook of Macroeconomics*, Amsterdam: Elsevier, 3-64.

[60] **Tan, H. B. and Richard Ashley** (1999a): "On the Inherent Nonlinearity of Frequency Dependent Time Series Relationships," in (P. Rothman, ed.) *Nonlinear Time Series Analysis of Economic and Financial Data*, Kluwer Academic Publishers: Norwell, 129-142.

[61] **Tan, H.B. and Richard Ashley** (1999b): "Detection and Modeling of Regression Parameter Variation Across Frequencies with an Application to Testing the Permanent Income Hypothesis," *Macroeconomic Dynamics*, 3, 69-83.

[62] **Tasci, Murat and Randal Verbrugge** (2014): "How Much Slack is in the Labor Market? That Depends on What You Mean by Slack," *Economic Commentary*, 2014-21, pp. 1-6.

[63] **Taylor, John B.** (1993): "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy*, 93, p. 195-214.

[64] **Taylor, John B. and John C. Williams**(2011): "Simple and Robust Rules for Monetary Policy" in Benjamin M. Friedman and Michael Woodford, eds., *Handbook of Monetary Economics* vol. 3B, North Holland, 829-60.

[65] **Woodford, Michael** (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

Figure 1: Plots of the 3 Aggregate Frequency Components of Inflation Rate

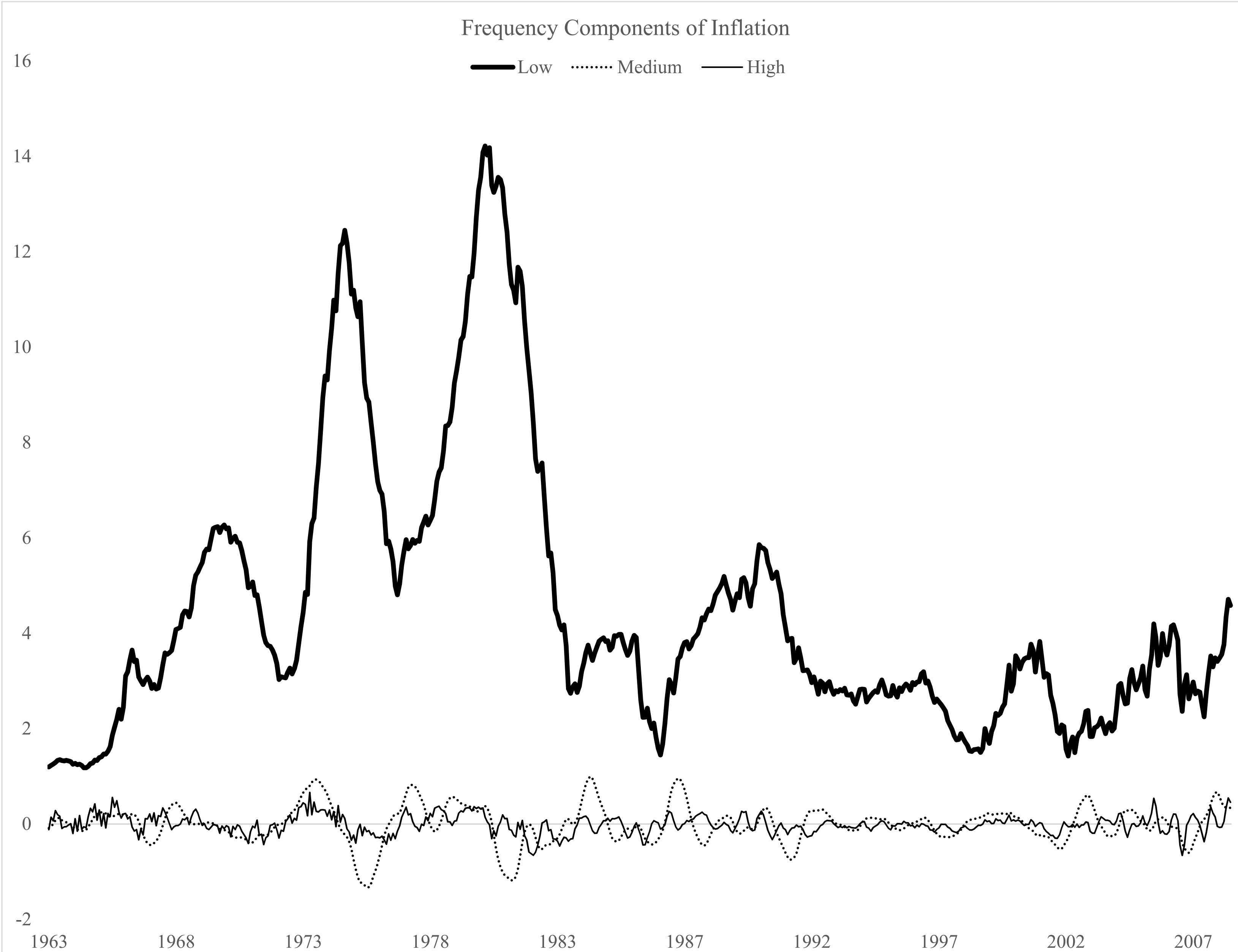
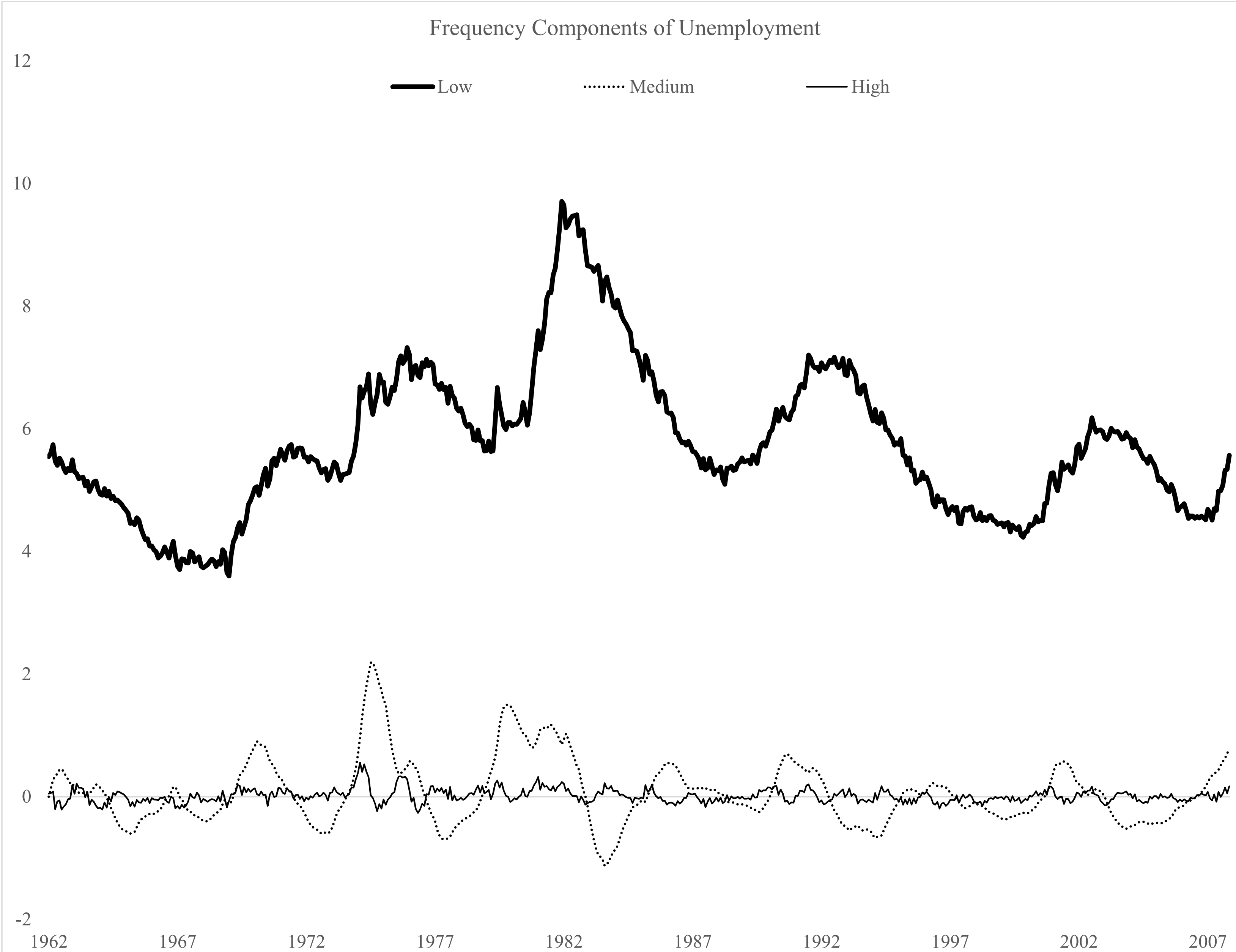


Figure 2: Plots of the 3 Aggregate Frequency Components of Unemployment Rate



Frequency Component	Frequency	Reversion Period[a]	Row Number(s) in A[b]
1	0	>36	1
2	$\pi/18$	$36/1 = 36.00$	2,3
3	$2\pi/18$	$36/2 = 18.00$	4,5
4	$3\pi/18$	$36/3 = 12.00$	6,7
5	$4\pi/18$	$36/4 = 9.00$	8,9
6	$5\pi/18$	$36/5 = 7.20$	10,11
7	$6\pi/18$	$36/6 = 6.00$	12,13
8	$7\pi/18$	$36/7 = 5.14$	14,15
9	$8\pi/18$	$36/8 = 4.50$	16,17
10	$9\pi/18$	$36/9 = 4.00$	18,19
11	$10\pi/18$	$36/10 = 3.60$	20,21
12	$11\pi/18$	$36/11 = 3.27$	22,23
13	$12\pi/18$	$36/12 = 3.00$	24,25
14	$13\pi/18$	$36/13 = 2.77$	26,27
15	$14\pi/18$	$36/14 = 2.57$	28,29
16	$15\pi/18$	$36/15 = 2.40$	30,31
17	$16\pi/18$	$36/16 = 2.25$	32,33
18	$17\pi/18$	$36/17 = 2.12$	34,35
19	$18\pi/18$	$36/18 = 2.00$	36

Table 1: **Frequencies and Reversion Periods for a 36-Month Window** [a]In months, calculated as 2π divided by the frequency. The sinusoids comprising the elements of the row(s) of the A matrix corresponding to this reversion period complete a full cycle in this many months. Thus, the scalar product of such a row with a time-series vector whose fluctuations self-reverse substantially slower than this will be very small. [b]The A matrix is defined in Equation (3).

Period	Matrix A										Data
> 10	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	$X_j(21)$
10	0.45	0.36	0.14	-0.14	-0.36	-0.45	-0.36	-0.14	0.14	0.36	$X_j(22)$
10	0.00	0.26	0.43	0.43	0.26	0.00	-0.26	-0.43	-0.43	-0.26	$X_j(23)$
5	0.45	0.14	-0.36	-0.36	0.14	0.45	0.14	-0.36	-0.36	0.14	$X_j(24)$
5	0.00	0.43	0.26	-0.26	-0.43	0.00	0.43	0.26	-0.26	-0.43	$X_j(25)$
3.3	0.45	-0.14	-0.36	0.36	0.14	-0.45	0.14	0.36	-0.36	-0.14	$X_j(26)$
3.3	0.00	0.43	-0.26	-0.26	0.43	0.00	-0.43	0.26	0.26	-0.43	$X_j(27)$
2.5	0.45	-0.36	0.14	0.14	-0.36	0.45	-0.36	0.14	0.14	-0.36	$X_j(28)$
2.5	0.00	0.26	-0.43	0.43	-0.26	0.00	0.26	-0.43	0.43	-0.26	$X_j(29)$
2	0.32	-0.32	0.32	-0.32	0.32	-0.32	0.32	-0.32	0.32	-0.32	$X_j(30)$

Table 2: **An Example With a Window of Length Ten Periods**. The first row of A time the data vector simply yields $1/\sqrt{T}$ times the sample mean of the data in this ten-period window. As the window moves through the data set, this operation extracts any, possibly nonlinear, trend as a moving average. Rows two and three take a weighted average of the window data, using smoothly-varying weights which take a full ten periods to reverse, so any fluctuation in window data that reverses in a couple of periods yields a small value. The product of row ten and the window data is essentially calculating five changes in the data which occur during the window period. A long, smooth variation in the window data yields a small value for this frequency component.

Sample	δ_1	δ_2	ϕ_π	ϕ_u	$\bar{R}^2$
MBM Period	0.578 (0.089)	0.246 (0.081)	0.702 (0.163)	-0.913 (0.289)	0.893
VGB Period	0.611 (0.126)	0.292 (0.116)	1.420 (0.497)	0.202 (0.505)	0.912

Table 3: **Estimates for the Non-Frequency-Dependent Taylor-type Model: Equation (2) of Section 2.2.** Nonlinear least squares estimates are quoted for Equation (2), with White-Eicker standard errors in parentheses. The estimates for the intercept term and the dummy variables used to eliminate the three outliers are not quoted.

		Full Disaggregation		Polynomial Smoothed		Three-Band Model	
		MBM Period	VGB Period	MBM Period	VGB Period	MBM Period	VGB Period
		estimate (*std.error*)	*estimate* (*std.error*)	*estimate* (*std.error*)	*estimate* (*std.error*)	*estimate* (*std.error*)	*estimate* (*std.error*)
δ_1		1.239 (0.095)	1.261 (0.075)	1.321 (0.095)	1.298 (0.071)	1.336 (0.099))	1.307 (0.070)
δ_2		-0.381 (0.096)	-0.347 (0.076)	-0.446 (0.095)	-0.372 (0.072)	-0.451 (0.098)	-0.375 (0.072)
$p\ \ (H_0 : \delta_1 = \delta_2 = 0\ )$		0.000	0.000	0.000	0.000	0.000	0.000
	Period (months)						
$\phi_{\pi,1}$	> 36	0.883 (0.093)	1.835 (0.270)	0.748 (0.117)	0.826 (0.309)	0.674 (0.149)	1.942 (0.332)
$\phi_{\pi,2}$	36	0.376 (0.891)	1.563 (1.802)	1.261 (0.379)	-1.200 (0.712)		
$\phi_{\pi,3}$	18	1.160 (0.941)	-1.440 (1.528)	1.713 (0.652)	-2.733 (1.243)	0.902 (0.719)	0.478 (1.393)
$\phi_{\pi,4}$	12	1.693 (1.936)	-5.236 (3.148)	2.106 (0.841)	-3.774 (1.632)		
$\phi_{\pi,5}$	9.0	0.421 (1.736)	-7.155 (2.001)	2.438 (0.967)	-4.323 (1.893)		
$\phi_{\pi,6}$	7.2	-0.238 (2.220)	-2.836 (2.712)	2.710 (1.075)	-4.379 (2.071)		
$\phi_{\pi,7}$	6.0	0.395 (3.597)	-3.369 (4.755)	2.923 (1.227)	-3.943 (2.242)	2.272 (1.573)	-6.040 (2.604)
$\phi_{\pi,8}$	5.1	3.130 (2.520)	-3.496 (2.325)	3.075 (1.492)	-3.014 (2.512)		
$\phi_{\pi,9}$	4.5	-0.659 (3.323)	-7.567 (3.861)	3.167 (1.911)	-1.593 (2.987)		
$\phi_{\pi,10}$	≤ 4.0	1.512 (2.096)	-2.721 (4.774)	3.199 (2.493)	0.320 (3.728)		
$\phi_{u,1}$	≥ 120	0.358 (0.200)	-0.070 (0.441)	-0.244 (0.162)	0.272 (0.210)	0.038 (0.104)	0.169 (0.240)
$\phi_{u,2}$	60	-3.290 (0.919)	-3.513 (2.934)	-0.890 (0.359)	-2.337 (0.707)		
$\phi_{u,3}$	40	-2.679 (1.098)	-7.500 (6.710)	-1.564 (0.538)	-4.703 (1.217)	-1.941 (0.729)	-2.338 (0.872)
$\phi_{u,4}$	30	-1.894 (1.151)	-36.189 (12.242)	-2.268 (0.732)	-6.823 (1.557)		
$\phi_{u,5}$	24	-1.516 (1.181)	-8.396 (13.805)	-3.001 (0.979)	-8.700 (1.826)		
$\phi_{u,6}$	20	-3.403 (1.942)	-52.189 (19.956)	-3.764 (1.313)	-10.332 (2.183)		
$\phi_{u,7}$	17.1	-3.197 (8.431)	-16.692 (22.387)	-4.556 (1.748)	-11.720 (2.802)	-5.490 (2.839)	-21.023 (4.702)
$\phi_{u,8}$	15	-13.557 (4.950)	-8.752 (18.034)	-5.378 (2.292)	-12.864 (3.779)		
$\phi_{u,9}$	13.3	-11.405 (4.780)	-62.290 (23.682)	-6.229 (2.947)	-13.763 (5.129)		
$\phi_{u,10}$	≤ 12	-2.775 (2.854)	-38.180 (29.569)	-7.109 (3.713)	-14.418 (6.837)		
Testing for Zero Coefficients							
$p\ \ (H_0 : \phi_{\pi,j} = 0\ \forall j)$		0.000	0.000	0.000	0.010	0.000	0.000
$p\ \ (H_0 : \phi_{u,j} = 0\ \forall j)$		0.001	0.000	0.016	0.000	0.000	0.000
Testing for Frequency Dependence							
$p\ \ (H_0 : \phi_{\pi,j} = \phi_{\pi,k}\ \forall j \neq k)$		0.742	0.000	0.209	0.023	0.624	0.007
$p\ \ (H_0 : \phi_{u,j} = \phi_{u,k}\ \forall j \neq k)$		0.000	0.000	0.006	0.000	0.006	0.000
$\bar{R}^2$		0.978	0.988	0.978	0.987	0.977	0.987

Table 4: **OLS Estimates for the Frequency-Dependent Taylor Rule: Equation(3) of Section 2.3.** White-Eicker standard errors are quoted in parentheses. The estimate for the intercept term and the dummy variables used to eliminate the three outliers are not quoted. The unemployment rate component with a reversion period of 120 months is included in the "Low-Band" rather than in the "Mid-Band" in the columns for the Three-Band Model.

Sample	δ_1	δ_2	ϕ_π	ϕ_u	$\bar{R}^2$
Sep 1987 - Dec 1994	1.18 (0.11)	-0.27 (0.10)	-0.12 (0.39)	-2.73 (0.57)	0.994
Jan 1995 - Dec 2000	1.13 (0.14)	-0.20 (0.14)	0.82 (0.32)	0.25 (0.41)	0.933

Table 5: **Estimates for the Non-Frequency-Dependent Taylor-type Rule: 'Old' Versus 'New' Economy**. Nonlinear least squares estimates are quoted for Equation (2), using the Ball and Tchaidze (2002) sub-periods; see discussion in Section 2.4. White-Eicker standard errors are quoted in parentheses.

		'Old' Economy		'New' Economy	
		estimate	*std.error*	*estimate*	*std.error*
δ_1		1.12	0.06	0.97	0.12
δ_2		-0.31	0.08	-0.18	0.10
	Period (months)				
$\phi_{\pi,\,low}$	≥ 36	0.24	0.25	0.65	0.11
$\phi_{\pi,\,medium}$		-0.18	0.39	-2.29	0.52
$\phi_{\pi,\,high}$	< 4	-0.31	0.08	-0.19	0.10
$\phi_{u,\,low}$	≥ 120	-2.44	0.27	0.26	0.14
$\phi_{u,\,medium}$		-1.33	0.28	-1.03	0.43
$\phi_{u,\,high}$	< 4	-2.28	1.87	1.75	1.41
Testing for Zero Coefficients					
$p\ \ (H_0 : \phi_{\pi,j} = 0\ \forall j)$		0.668		0.000	
$p\ \ (H_0 : \phi_{u,j} = 0\ \forall j)$		0.000		0.000	
Testing for Frequency Dependence					
$p\ \ (H_0 : \phi_{\pi,j} = \phi_{\pi,k}\ \forall j \neq k)$		0.046		0.000	
$p\ \ (H_0 : \phi_{u,j} = \phi_{u,k}\ \forall j \neq k)$		0.016		0.000	

Table 6: **Test of Taylor-type Rule Policy Unresponsiveness in the 'Old' Versus the 'New' Economy, Allowing for Frequency Dependence in the Relationship.** Rejection p-values for the null hypothesis that the coefficients on the three aggregate frequency components for inflation or unemployment rate are all zero. These are results from re-estimation of Equation (3), using the Ball and Tchaidze (2002) sub-periods; see discussion in Section 2.4.

	Conventional Moving Average u_t^*		Frequency Component u_t^*	
	estimate	*std.error*	*estimate*	*std.error*
δ_1	1.33	0.07	1.33	0.07
δ_2	-0.40	0.07	-0.39	0.07
π_t^*	1.41	0.31	1.76	0.37
$\pi_t - \pi_t^*$	-0.84	0.75	-1.38	1.38
u_t^*	0.94	0.26	0.11	0.26
$u_t - u_t^*$	-0.45	0.23	-2.41	0.99

Table 7: **Comparison of An Estimated Policy Rule Based on a Frequency-Component-Based Natural Unemployment Rate Decomposition Versus One Based on a Conventional (Moving Average) Decomposition.** These estimation results compare an estimated two-band federal funds rate policy equation using our implicit estimate of the natural rate of unemployment (based on the low-frequency component of the unemployment rate) to an analogous policy equation estimated using a conventional one-sided moving average decomposition of the unemployment rate. The variable u_t^* is an estimate of the natural rate of unemployment obtained from either a conventional one-sided moving average or from the lowest frequency component of the unemployment rate; the variable π_t^* is the lowest frequency component of the inflation rate in both models. See discussion in Section 2.5.

4 Technical Appendix

4.1 Modeling Frequency Dependence

In this section we discuss the technique used here for modeling frequency dependence in the monetary policy rule, Equation (2) above.[26]

The idea of regression in the frequency domain can be traced back to Hannan (1963) and Engle (1974, 1978), and is further developed in Tan and Ashley (1999a and 1999b), who developed a real-valued reformulation of Engle's (1974) complex-valued framework.

Consider the ordinary regression model:

$$Y = X\beta + e \quad e \sim N(0, \sigma^2 I) \tag{4}$$

where Y and e are each $T \times 1$ and X is $T \times K$. Now define a $T \times T$ matrix A, whose $(s,t)^{th}$ element is given by:

$$a_{s,t} = \begin{array}{l} (\frac{1}{T})^{\frac{1}{2}} \quad \text{for } s = 1; \\ (\frac{2}{T})^{\frac{1}{2}} \cos(\frac{\pi s(t-1)}{T}) \quad \text{for } s = 2, 4, 6, ..., (T-2) \text{ or } (T-1); \\ (\frac{2}{T})^{\frac{1}{2}} \sin(\frac{\pi (s-1)(t-1)}{T}) \quad \text{for } s = 3, 5, 7, ..., (T-1) \text{ or } T; \\ (\frac{1}{T})^{\frac{1}{2}} (-1)^{t+1} \quad \text{for } s = T \text{ when } T \text{ is even.} \end{array} \tag{5}$$

It can be shown that A is an orthonormal matrix, so its transpose is its inverse and Ae is still distributed $N(0, \sigma^2 I)$. Pre-multiplying the regression model (4) by A thus yields,

$$AY = AX\beta + Ae \rightarrow Y^* = X^*\beta + e^*, e^* \sim N(0, \sigma^2 I) \tag{6}$$

where Y^* is defined as AY, X^* is defined as AX, and e^* is defined as Ae. The dimensions of the of Y^*, X^*, and e^* arrays are the same as those of Y, X, and e in Equation (4), but the T components of Y^* and e^* and the rows of X^* now correspond to frequencies instead of time periods.

To fix ideas, we initially focus on the j^{th} component of X, i.e., column j of the X matrix, corresponding to the $j-1^{st}$ explanatory variable if there is an intercept in the model. The T frequency components are partitioned into M frequency bands, and M $T \times 1$ dimensional dummy variable vectors, $D^{*1}, ..., D^{*M}$, are defined as follows: for elements that fall into the s^{th} frequency band, $D^{*s,j}$ equals $X^*_{\{j\}}$, and the elements are zero otherwise. The regression model

[26] See Ashley and Verbrugge (2009) for details; this section provides the most up-to-date exposition, however. Additional descriptions are given in Ashley and Verbrugge (2007a,b), Ashley and Tsang (2013), and in Ashley and Li (2014).

can then be generalized as:

$$Y^* = X^*_{\{j\}}\beta_{\{j\}} + \sum_{m=1}^{M} \beta_{j,m} D^{*m,j} + e^* \tag{7}$$

where $X^*_{\{j\}}$ is the X^* matrix with its j^{th} column deleted and $\beta_{\{j\}}$ is the β vector with its j^{th} component deleted.

To test whether the j^{th} component of β is frequency-dependent (i.e., to test whether the effect of the j^{th} variable in X on Y is frequency or persistence dependent) one can then simply test the null hypothesis that $H_0 : \beta_{j,1} = \beta_{j,2} = ... = \beta_{j,M}$.

In the present application, we focus on two columns of X: the real-time inflation rate and the real-time inflation rate; these columns are denoted j and k below. By the same reasoning used above, one can quantify (and test for) frequency dependence in the two model coefficients β_j and β_k corresponding to these two columns by re-writing the regression model (Equation 7) as:

$$Y^* = X^*_{\{j,k\}}\beta_{\{j,k\}} + \sum_{m=1}^{M} \beta_{j,m} D^{*m,j} + \sum_{m=1}^{M} \beta_{k,m} D^{*m,k} + e^*. \tag{8}$$

To make this regression equation a bit more intuitive, one can back-transform Equation (8) back into the time domain by pre-multiplying both sides of this equation with the inverse of A, which (because A is an orthonormal matrix) is just its transpose:

$$A'Y^* = A'X^*_{\{j,k\}}\beta_{\{j,k\}} + A'\sum_{m=1}^{M} \beta_{j,m} D^{*m,j} + A'\sum_{m=1}^{M} \beta_{k,m} D^{*m,k} + A'e^*. \tag{9}$$

This yields the time-domain specification:

$$Y = X_{\{j,k\}}\beta_{\{j,k\}} + \sum_{m=1}^{M} \beta_{j,m} D^{m,j} + \sum_{m=1}^{M} \beta_{k,m} D^{m,k} + e. \tag{10}$$

where $X_{\{j,k\}}$ is the original X matrix, omitting columns j and k and $\beta_{\{j,k\}}$ is the original β vector, omitting components j and k

Note that now the dependent variable is the same time series (Y) as in the original model, Equation (4). Similarly, all of the explanatory variables – except for the j^{th} and k^{th} – are the same as in the original model. Indeed, the only difference is that these two explanatory variables have each been replaced by M new variables: i.e., the explanatory variable X_j has been replaced by $D^{1,j}...D^{M,j}$ and the the explanatory variable X_k has been replaced by $D^{1,k}...D^{M,k}$. Each of these M variables can be viewed as a bandpass-filtered version of the original data (the j^{th} or

k^{th}column of the X matrix), with the nice property that the M frequency component variables corresponding to column j of the X matrix add up precisely to the j^{th} column of X and the M frequency component variables corresponding to column k of the X matrix add up precisely to the k^{th} column of X.

In other words, the j^{th} column of X – for example – is now partitioned into M parts. Reference to definition of the A matrix in Equation (5) shows that the first (low-frequency) component (corresponding to $m = 1$, if the first band has only a single component) is proportional to the sample average of the data for this explanatory variable. Similarly, the last component (corresponding to $m = M$, if the last band has only a single component – as it will whenever M is an even number) is essentially a sequence of changes in the data, and hence is the highest-frequency component that can be extracted from the data on this variable. To test for frequency dependence in the regression coefficient on this j^{th} regressor, then, all that one need do is test the joint null hypothesis that $\beta_{j,1} = \beta_{j,2} = ... = \beta_{j,M}$. Similarly, X_k is replaced by $D^{1,k}...D^{M,k}$ and one tests the null hypothesis that $\beta_{k,1} = \beta_{k,2} = ... = \beta_{k,M}$.

However, because the A transformation mixes up past and future values (as in any Fourier-based bandpass filter), it can be shown that these M frequency components are correlated with the model error term e if there is feedback between Y and either of these two explanatory variables, leading to inconsistent estimation of the parameters $\beta_{j,1}, \beta_{j,2}, ...\beta_{j,M}$ and $\beta_{k,1}, \beta_{k,2}, ...\beta_{k,M}$ in that case. Feedback between the federal funds rate and inflation or unemployment rates is certainly likely, so this is an important issue here. To avoid this problem in general, Ashley and Verbrugge (2009) suggest modifying the procedure described above in order to obtain a one-sided filter for partitioning a variable into its frequency components. In particular, they suggest decomposing X_j, the j^{th} explanatory variable data vector, into frequency components by applying the transformation described above within a moving window, retaining only the most recent frequency component values calculable from this window. Since the discussion from this point forward entirely focuses on extracting frequency component from such moving windows, the symbol T_{full} will henceforth be used to denote the length of the full sample, as this is now distinct from dimensionality (T) of the A matrix for each window. T_{full} is a constant – that is, the windows do not increase in length as they move through the sample.

The filtering of X_k is similar to that for X_j, but we note that there is no need to constrain the window length (T_{full}) to be identical for both variables: this is a modeling decision which should be based on relevant economic theory and on the data themselves. Indeed, we discuss this issue above (in Section 2.1) and do set distinct window lengths for use in obtaining the frequency components for each of the two explanatory variables in the Taylor Rule formulations considered here. In particular, we set a window length of $T_j \equiv 36$ months for decomposing the inflation rate data ("X_j") into its frequency/persistence components, and we set a window length of $T_k \equiv 120$ months for similarly decomposing the unemployment rate data. Thus, the transformation matrix A defined in Equation (5) is of dimension $T_j = 36 \times 36$ months for the inflation rate data and is of dimension $T_k = 120 \times 120$ months for the unemployment rate data. For expositional clarity, however, we focus almost entirely on the case $T_j = 36$ in the remainder of the present sub-section, and below.

It would, at the outset then, appear that the sample average over the T_j observations – corresponding to the zero-frequency component ($m = 1$), and obtained using the first row of the A matrix – thus becomes a moving average of order T_j once the filtering is now being applied to a moving window, so that this first component is now extracting a backward-looking nonlinear trend from the full sample of data, using a moving average of order T_j. It is well-known, however, that such moving average trend estimates have very poor properties. In particular, they are known to induce pronounced phase shifts in the estimated trend time series, which can substantially distort the apparent turning points in the time series. We consequently instead separately de-trend the data in each window prior to filtering – using a linear time trend regression – and (as with the other frequency components) use the most recent value of this component in the window as frequency component for the time period corresponding to the last time period in the window. Because of this window-specific de-trending, the resulting backward-looking non-linear trend time series (which then becomes the zero-frequency component for this time period in the full data set) is no longer equivalent to a one-sided moving average trend estimate and does not exhibit the concomitant phase and turning point distortions characteristic of such trend estimates.

This moving-window approach is used in the present paper for two additional, and here crucial, reasons. First, the moving window makes it possible – and, indeed, easy – to use real-time data for the values of X_j and X_k: the data used in each window is simply that available at the time period which is the window's endpoint. The frequency decomposition is in this way gracefully consistent in each period with the data which were available to the policymakers at the time. Second, the limited length of the window helpfully restricts the number of distinct frequency components mathematically allowable, concomitantly reducing the number of coefficients which need to be estimated in Equation (10). More explicitly, if one does *not* use moving windows (of length T_j in decomposing explanatory variable X_j and of length T_k in decomposing explanatory variable X_k), then this is equivalent to using a single window (of length T_{full}) in decomposing both of these variables. In that case – presuming that T_{full} is an even number – then reference to Table 1 (in which T_{full} is 36) makes it evident that there will be $\frac{T_{full}}{2}+1$ distinct frequency components – and hence $\frac{T_{full}}{2}+1$ coefficients to estimate in Equation (10) – for each of the two variables. Clearly, it is infeasible to estimate all of the coefficients when more than one explanatory variable is being decomposed and desirable to use multiple (moving) windows of length substantially smaller than the full sample length, T_{full}.

Restricting attention, for expositional clarity, to the windowing for variable X_j, we see from Table 1 that setting the length of the window to $T_j = 36$ implies that that the matrix A defined in Equation (5) involves only 19 distinct frequencies. Thus, the value of M in the weighted sum of the $D^{m,j}$ in Equation (10) is only 19, so that Equation (10) involves the estimation of only 18 additional coefficients in order to model any frequency dependence in the relationship between Y and X_j.[27] (On the other hand, the lowest frequency – longest period – component of X_j cannot distinguish between fluctuations with a reversion period of 36 months and fluctuations with reversion periods longer than this.) Section 4.3, below, illustrates this calculation of the number of distinct frequencies in some detail – and also supplies more of the intuition behind the frequency components – for the special case where the window length is set (for expositional simplicity) to just 10; Table 1 tabulates the 19 frequency components possible with a the 36-month window length used here, explicitly relating each one to its reversion period and to the corresponding row (or rows) of the A matrix.

Thus, noting that a 120-month window length is chosen (in Section 2.1) for decomposing X_k into frequency components, it is entirely feasible in this moving-window framework to estimate all of the possible distinct coefficients – i.e., $\beta_{j,1}, \beta_{j,2}, \cdots \beta_{j,19}$ and $\beta_{k,1}, \beta_{k,2}, \cdots \beta_{k,61}$ – without aggregating the frequency components into selected frequency bands. This choice – to not aggregate any of the components – is not necessarily optimal, however, because the estimation of $19+61 = 80$ coefficients can use up so many degrees of freedom in the dataset as to adversely, and needlessly, impact the precision with which inferences can be made. Moreover, reference to Table 1 makes it clear that, although the frequencies associated with these 19 components are equally spaced on the interval from zero to π, the corresponding reversion periods (which are proportional in each case to the reciprocal of the frequency) are quite unevenly spaced. In particular, we note that the eight components corresponding to $\beta_{j,11}, \cdots \beta_{j,19}$ all have reversion periods of between two and three months. While it is an empirical issue whether or not the FOMC pays any attention to π_t or u_t fluctuations this brief, we find that it is not useful to estimate 9 additional coefficients for the π_t components and 51 additional coefficients for the u_t components so as to quantify the degree to which the FOMC distinguishes between fluctuations with periods between two and four months in length. For this reason we routinely aggregate the nine highest frequency components for π_t and the 51 highest frequency components for u_t into two components; this is, in effect, assuming that $\beta_{j,10}, \cdots \beta_{j,19}$ are all essentially equal and that $\beta_{k,10}, \cdots \beta_{k,61}$ are all essentially equal. Note that this is why M equals 10 in both of the sums

[27] This total of 19 distinct frequencies includes the 36-month moving average formed by the first row of A (corresponding to a frequency of zero) and 18 distinct positive frequencies, corresponding to the remaining rows of A, taken pairwise where the sine and cosine yield distinct rows, with the trend value added to the zero-frequency term in a separate step, as indicated above.

appearing in Equation (3), and this is reflected in the number of estimation entries appearing in Table 4.

In addition, one might be interested in even more highly aggregated frequency bands, so as to facilitate exposition of one's results on economic or intuitive grounds, as in Ashley and Tsang (2013) and Ashley and Li (2014), where the frequency components are aggregated into just three bands. In Section 2.1 w do the same, defining a low frequency band ("Low-Band") corresponding to fluctuations with reversion periods in excess of 36 months, a medium frequency band ("Mid-Band")with reversion periods between 12 and 36 months, and a high frequency band ("High-Band") corresponding to reversion periods of less than 12 months. Referring to Table 1 (with regard to the 36-period windows for the π_t data) and to Table 4, the "Low-Band" consists of the first (nonlinear trend) component; the "Mid-Band" is the sum of the components for reversion periods of 12, 18, and 36 months; and the "High-Band" comprises the sum of the remaining components, which all revert more quickly. Yes, this particular partitioning of the 19 components (of π_t) or the 61 components (of u_t) into just three bands is, in a sense, a bit arbitrary. On the other hand, these three aggregated bands are interpretable in terms of the roughly the same calendar that the FOMC's policymakers live on.

Lastly, when decomposing X_j using a window, one must confront the problem of "edge effects" near the window endpoints. As in Dagum (1978) and Stock and Watson (1999), this problem is dealt with by augmenting the sample data used in each window with projected data, here with 6 months (in the 36-month π_t windows) or with 24 months (in the 120-month u_t windows). Thus, a 36-month window incorporating the real-time data on X_j as of period t includes 30 past values of X_j – as known at time t – plus projections (forecasts) of its values for months $t+1$ to $t+6$. This window of data is then used, as described above, to compute the corresponding $M = 19$ components of X_j – i.e., the vectors $D^{1,j}...D^{19,j}$. The 30^{th} element of each of these 36-vectors is then used as the period-t filtered value of X_j for this particular frequency. We find that the estimated values of the coefficients $(\beta_{j,1}, \beta_{j,2}, ...\beta_{j,19})$ on $D^{1,j}...D^{19,j}$ (which are the 19 frequency components of π_t) and their estimated standard errors are not sensitive to the number of projection periods (as long as at least 6 months of projections are used), nor to the details of how the projections (forecasts) are produced. Similarly, we find that the estimated values of the coefficients $(\beta_{k,1}, \beta_{k,2}, ...\beta_{k,61})$ on $D^{1,k}...D^{61,k}$ (which are the 61 frequency components of u_t) and their estimated standard errors are also not sensitive to the number of projection periods (as long as at least 12 months of projections are used), nor to the details of how the projections (forecasts) are produced.[28] Since it is well known that the FOMC

[28]The empirical results quoted in Section 2 above use projections based on an AR(4) model, estimated using the (real-time) sample data in the window. Similar results, obtained using more complex projection models and/or 48-month windows are available from the authors. Also available are both RATS code and a ready-to-use Windows-based executable file implementing this decomposition method. All that the user needs to do is specify

makes extensive use of forecasts in its decision-making, utilizing projections of this nature in our windowed bandpass filters seems particularly appropriate.

This is a good point at which to contrast the frequency decomposition used here with superficially similar procedures in the existing literature. For example, in contrast to trend-cycle decomposition methods (e.g., Beveridge-Nelson), our approach does not decompose an explanatory variable like X_j into just two components: an arbitrarily-persistent $I(1)$ or $I(1)$-like trend and a stationary $I(0)$ fluctuation. Our decomposition instead produces M components (adding up to X_j) which span the complete range of persistence levels permitted by the chosen window length. And it allows the data itself – via regression analysis applied to Equation (10), or the closely related Equation (3) – to quantify how the coefficients $\beta_{j,1}, ..., \beta_{j,M}$ vary across all of these persistence levels. Further, our decomposition still yields consistent parameter estimation where (as is typically the case with economic relationships) one cannot rule out feedback (or bi-directional causality); this is in contrast to the earlier spectral regression models cited at the outset of this section, which employ two-sided filtering and yield inconsistent parameter estimation in the presence of feedback. Finally, our approach is uniquely appropriate to the present analysis of the FOMC's Taylor Rule behavior, because the central bank surely bases its actual policy decisions on real-time data. In particular, the current real-time history of each of the relevant explanatory variables (the inflation and unemployment rates) corresponds exactly to the data which we use in each window for the decomposition of the current value of each variable into its frequency/persistence components.

We note, in this context, that an analogous kind of analysis based on the gain and phase of a transfer function model for the federal funds rate – as in Box and Jenkins (1976, Part III) – would be problematic because such models characteristically involve lagged values of the dependent and explanatory variables. For one thing, models containing lagged variables are inherently awkward when using real-time data because it is not clear whether the period-t datum to be used for X_j lagged, say, two periods should be the value of for that period as known currently (i.e., in period t) or at the time (i.e., in period $t-2$). In addition, transfer function gain and phase plots are substantially more challenging to interpret than our $\beta_{j,1}, ..., \beta_{j,M}$ coefficients, especially where (as here) bi-directional causality is likely. For example, Granger (1969) notes, "in many realistic economic situations, however, one suspects that feedback is occurring. In these situations the coherence and phase diagrams become difficult or impossible to interpret, particularly the phase diagram."

the order of the AR(p) model, the number of projections, and the window length; the software then produces the full allowed number of frequency components and the user can aggregate these components into frequency bands (per the previous paragraph) as desired.

4.2 The Appeal of this Frequency-based Approach to Disaggregation by Persistence Level

The focus of this paper is to investigate, in a data-driven way, the degree and manner to which the FOMC has responded to persistent innovations in the unemployment rate and the inflation rate differently than it has to more transitory fluctuations in those variables. Thus the objective of partitioning two of the explanatory variable time series – X_j and X_k in Section 4.1 above, which are the real-time unemployment and inflation rates in the present application – is not the bandpass filtering *per se*. Rather, we decompose the unemployment and inflation rates into frequency components so that we can separately estimate the impact of fluctuations of distinctly different persistence levels in these two variables on the federal funds rate and make inferences concerning these differential impacts; this allows a richer consideration of how FOMC policy has varied over the several time periods considered.

No representation is made here that the bandpass filtering described in Section 4.1 above is asymptotically optimal – e.g., as in Koopmans (1974) or Christiano and Fitzgerald (2003) – although the relevance of asymptotic optimality in filtering windows of data which here can be of sample length ca. 36 months is debatable.[29] On the other hand, our method of decomposing a time series into M frequency components has several very nice characteristics, which make this decomposition approach overwhelmingly well-suited to the present application:

1) The M frequency components that are generated from an explanatory variable (i.e., from a column of X) by construction partition it. That is, these M components add up precisely to the original observed data on this column of X. This makes estimation and inference with regard to frequency dependence (or its inverse, persistence dependence) in the corresponding regression coefficient particularly straightforward: we simply replace this explanatory variable in the regression model by a linear form in the M components and analyze the resulting M coefficient estimates.

2) Due to the moving windows used, this particular way of partitioning the data on an explanatory variable into these M frequency components by construction utilizes backward-looking (i.e., one-sided) filters. As demonstrated in Ashley and Verbrugge (2009b), this feature is crucial to consistent OLS coefficient estimation where there is bi-directional Granger-causality (i.e., feedback) between the dependent variable and the explanatory variable being decomposed by

[29] In this context we note that it feasible – albeit somewhat awkward – to iteratively employ a Christiano-Fitzgerald (2003) low-pass filter to partition the data in such a way that the frequency components still add up to the original data. This procedure involves applying the filter repeatedly, at each iteration varying the frequency threshold and applying the filter to the residuals from the previous iteration. This procedure is, of course, no longer even asymptotically optimal, but it does yield frequency components which still add up to the original data – as ours do automatically. Experiments with decompositions along these lines did not yield noticeably distinct results with regard to inferences on the regression model coefficients.

frequency. The dependent variable in the present context is the federal funds rate, which is quite likely to be in a feedback relationship with the unemployment and inflation rates.
3) Finally, this way of partitioning the data on an explanatory variable into frequency/persistence components is not just mathematically valid and straightforward, it is also intuitively appealing. In particular – in contrast to many analyses in the frequency domain – our decompositions are not a 'black box.' The next section illustrates this point with a simple example.

4.3 An Illustrative Example with a Very Short Window

An example with a window ten periods in length illustrates the sense in which the frequency components defined above are extracting components of, say, X_j of differing levels of persistence. This window length is sufficient large as to illustrate the point, while sufficiently small as to yield an expositionally manageable example.[30] In particular, Table 2 displays the multiplication of the matrix A – whose elements are defined in Equation (5) – by the ten-component sub-vector of X_j corresponding to a window beginning in the particular period 21 and ending in period 30.

The first row of the A matrix is just a constant. The operation of this row of A on this particular ten-dimensional sub-vector of X_j is just calculating the sample mean over these ten observations. Thus, as this window progresses through the entire sample of data X_j, the first component of the vector formed by multiplying each ten-dimensional sub-vector of X_j on the left by A represents a one-sided, real-time, nonlinear trend estimate based (in this example) on a 10-period moving average.[31] This is the "zero-frequency" component of the full X_j vector, corresponding to a sinusoidal reversion period unbounded in length. This component of X_j includes all of its variation at frequencies so low (i.e., reversion periods so large) that they are essentially invisible in a window which is only ten periods in length.

Higher-frequency components of X_j are, conversely, distinguishable using this window. The "Period" column in Table 1 is the number of observations over which the sine or cosine used in the corresponding row of the A matrix completes one full cycle. This is ten observations for rows two and three of this A matrix, $\frac{10}{2} = 5$ observations for rows four and five, $\frac{10}{3} = 3\frac{1}{3}$ observations for rows six and seven, $\frac{10}{4} = 2\frac{1}{2}$ observations for rows eight and nine, and $\frac{10}{5} = 2$ observations for

[30] As described above, the empirical implementation in this paper uses a window 36 months in length (for π_t) and 120 months in length (for u_t). See Table 1 for an explicit listing of the component frequencies, the corresponding reversion periods, and the corresponding A matrix rows for a 36-dimensional A matrix.

[31] As noted in Section 2.1 above, simple moving average trend estimators are unsatisfactory, in that they (for example) yield trend estimates with substantially distorted turning points. For this reason we additionally estimate a linear time trend in each window in obtaining the frequency components for u_t and π_t in the empirical work here, as noted in Section 4.1 above: this complication is suppressed in the present sub-section so as to focus attention on the A matrix in a setting so simple as to elucidate how the application of this matrix is extracting components for which the terms "frequency" and "reversion period" are intuitively meaningful verbal constructs.

row ten.[32] In the most common convention, the frequency is defined as $\frac{\pi}{2}$ times the inverse of cycle length (period) of the corresponding sine or cosine for that row of the A matrix, in which case the frequencies run from zero (for row one) to π for row ten.

To see intuitively why multiplication of the X_j vector by, for example, rows two and three extract only slowly-varying fluctuations in X_j, notice that these two rows are smoothly varying weights that will be applied to the ten components of X_j in forming its dot (or scalar) products with these two rows. Slowly-varying fluctuations in X_j will thus have a large impact on these two dot products, whereas rapidly-reverting variations in X_j will have little effect on the values of these two dot products. Hence, components two and three of the matrix product AX_j will 'contain' only those parts of X_j which are slowly varying.

Conversely, it is evident upon inspection of the last row of the A matrix that only high-frequency fluctuations – i.e., fluctuations which reverse in just two months or so – will contribute significantly to the tenth component of AX_j.

Thus, the first rows of the A matrix are distinguishing and extracting what are sensibly the "low-frequency" or "large period"or "highly persistent" or "relatively permanent" components of this ten-month X_j sub-vector as the window moves through the sample. Conversely, the last rows of the A matrix are distinguishing and extracting what are sensibly the "high-frequency" or "small period"or "low persistence" or "relatively temporary" components of this X_j sub-vector.

[32]The number of observations in the sub-vector is an even integer – ten – in this example, implying that the sine and cosine terms are multiples of one another for what becomes a singleton last (tenth) row of the A matrix.