Are Technology Shocks Nonlinear?

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Abstract

This paper examines the behavior of postwar real U.S. GNP, the inputs to an aggregate production function, and the associated Solow residuals for the presence of nonlinearities in their generating mechanisms. To test for nonlinearity, we implement three different statistical tests: the McLeod-Li test based on the correlogram of the squared data, the BDS test based on the correlation integral, and the Hinich bicovariance test based on the third-order moments of the data.

We find substantial evidence that the generating mechanism of real GNP growth is nonlinear, but no evidence for nonlinearity in the Solow residual generated under a variety of specifications corresponding to different assumptions about market structure and about the measurement of inputs. Our results imply that it is the macroeconomy itself which is nonlinear – not the factor productivity or technology shocks impinging upon it. We further find that the generating mechanism of the labor input series (expressed as hours worked) is nonlinear whereas that of the capital services input (expressed several ways) appears to be linear. Consequently, we conclude that the source of the nonlinearity observed in real output is in the labor markets rather than in exogenous technology shocks.

1 Introduction

A recent strand of the macroeconomics literature has sought to explain the behavior of key economic series in terms of nonlinear time series models. Notable among these analyses is Neftçi (1984), who models asymmetries in the cyclical behavior of the U.S. unemployment rate using a discrete Markov process. Other examples include Stock (1987) and Hamilton (1989), who propose nonlinear statistical models to describe the behavior of such series as output, unemployment, etc. while Hinich and Patterson (1985), Brock and Sayers (1988), and Ashley and Patterson (1989) test for nonlinearity in these series directly. Since a number of papers have found that the generating mechanism for real output is nonlinear and nonlinear in an asymmetric way (see, for example, Blatt (1978), Neftçi (1984), Hamilton (1989), Ashley and Patterson (1989), and Potter (1995) as well as our own results reported below), it is of interest to determine the source of this nonlinearity.

In this paper, we model the behavior of real output in terms of an aggregate production function and test for nonlinearities in the generating mechanisms for real output growth, for its observable determinants (measures of the labor and capital inputs), and for an exogenous technology shock quantified by the Solow residual implied by this specification. In Section 2 we discuss the generation of the Solow residual under the standard assumptions of an aggregate constant returns to scale production technology and perfect competition; for reasons discussed later in this Section we repeat the analysis in Section 5 using Solow residuals computed under alternative assumptions that allow for variable factor utilization rates and for imperfect competition in product markets.

What is meant by the term "nonlinear generating process" used above? Any linear, nondeterministic, covariance stationary process can be written as the convolution of a weighting function with an identically and independently distributed noise process. If this noise process is gaussian, then the linear process will also be gaussian. In discrete time, this convolution can be written as:

$$x_t = \alpha + \sum_{k=0}^{\infty} \psi_k \epsilon_{t-k}; \tag{1.1}$$

where the sequence $\{\epsilon_t\}$ is the noise process and $\{\psi_k\}$ is the weighting function. Put another way, $\{\epsilon_t\}$ can be regarded as the input to a linear filter with impulse response weights $\{\psi_k\}$.

The model (1.1) is nonlinear if the input noise process, $\{\epsilon_t\}$, alters the impulse response weights, $\{\psi_k\}$. More explicitly, let $\{\epsilon_M\}$ be a sequence of past values of the noise process up to and including lag M. Then the impulse response weights could be expressed as:

$$\psi_k(\epsilon_M) = \psi_k + \sum_{l=0}^{M} \beta(l,k)\epsilon_{t-l-k}, \tag{1.2}$$

where $\beta(l,k)$ is another weighting function that is convolved as shown with the input noise process. If the $\{\psi_k\}$ in (1.1) are replaced by the $\{\psi_k(\epsilon_M)\}$ given in (1.2), then the process $\{x_t\}$ is nonlinear even if the noise process $\{\epsilon_t\}$ is gaussian.

A simple example of a nonlinear process is a bilinear model of the type proposed by Granger and Anderson (1978):

$$x_t = \beta\epsilon_{t-1}x_{t-2} + \epsilon_t, \qquad \epsilon_t \sim iid(0,\sigma_\epsilon^2). \tag{1.3}$$

Since x_{t-2} can be expressed in terms of ϵ_{t-2}, ϵ_{t-3}, etc., model (1.3) is a special case of (1.1) and (1.2). The model (1.3) is nonlinear because the weight placed on ϵ_{t-1} in (1.3) depends on past values of the input noise process. Further, in the bilinear model of (1.3) the output process $\{x_t\}$ is serially uncorrelated; thus, the optimal linear forecast of x_t is zero. But $\{x_t\}$ generated by the nonlinear process (1.3) are clearly not serially independent; in fact, Granger and Andersen show that, for this generating mechanism, optimal forecasts of $\{x_t\}$ based on its own past can have a forecast error variance as much as 50% smaller than that of the linear forecast.

This distinction is potentially quite consequential since statistical inferences based on a structural model for x_t which is mistakenly specified to be linear could be seriously biased. For example, goodness-of-fit measures examining only various conditional and unconditional second moments may be inadequate in this context. Moreover, the linear (or log-linear) forecasting/decision rules typically used in modelling expectations formation in macroeconomic models are optimal ("rational") only in the context of a linear model.

Below we test for nonlinearity in the generating mechanisms of U.S. real output, the inputs to an aggregate production function, and several estimates of the Solow residual using three different statistical tests. These tests (described in more detail in Section 3) are:[1]

1. McLeod-Li test [McLeod and Li (1983)]

2. BDS test [Brock, Dechert, and Scheinkman (1987)]

[1] MS-DOS software implementing these tests is available from the authors as part of a "nonlinearity toolkit."

3. Hinich bicovariance test [Hinich (1995), Hinich and Patterson (1995)]

The McLeod-Li test is based on the sample correlogram of the square of the data. This test is examining selected fourth moments of the data; in essence, it is testing for conditional heteroscedasticity (ARCH) e®ects. The BDS test is based on a nonparametric measure of association between a time series and its recent past. Originally proposed as a test for deterministic chaos in economic time series, the BDS test is now typically applied to prewhitened data as a test for serial independence. The Hinich bicovariance test systematically examines third moments of the series; it is a time-domain analogue of the Hinich bispectral test. The Hinich bispectral test and the kinds of nonlinear generating mechanisms most amenable to detection by third-moment techniques are described in Hinich (1982), Hinich and Patterson (1985), Ashley, Hinich, and Patterson (1986), and Ashley and Patterson (1989).[2] The bicovariance test is used here in view of the small sample sizes available. Since all of these tests are valid only in large samples { very large samples in the case of the BDS test { the results presented below are all based on bootstrapped simulations.

Our ¯ndings have important rami¯cations from the point of current business cycle analysis because, as Cochrane (1994) acknowledges, linearity is a key assumption underlying the studies in this ¯eld. To uncover the importance of di®erent shocks, these studies either make use of the VAR (vector autoregression) approach proposed by Sims (1980), or use linear (or log-linear) laws of motion for the endogenous series as described, for example, by Kydland and Prescott (1982).[3] Following the Slutsky/Frisch tradition, these studies model cyclical °uctuations in terms of random impulses that are propagated by means of linear transmission mechanisms. However, if the generating mechanism for real output or other endogenous series such as hours worked is nonlinear, then the e®ects of alternative shocks may be misspeci¯ed.

Our results using Solow residuals based on the original growth accounting framework proposed by Solow (1957) are reported and discussed in Section 4. In Section 5 we extend our analysis to encompass more recent developments that have been discussed under the topic of the procylicality

[2]Third-moment techniques are sensitive to forms of nonlinearity that yield asymmetric time series; testing for asymmetry per se { as in Mittnik and Niu (1994), Ramsey and Rothman (1996), and Verbrugge (1996) { is beyond the scope of the present paper, however.

[3]Linear laws of motion have also been exploited in studies that use estimation rather than calibration. See, for example, Christiano (1988) or Altug (1989).

of productivity. According to proponents of the real business cycle approach (Prescott (1986), for example) the observed procyclical movements in productivity are a response to exogenous technology shocks. In a series of papers, Hall (1988, 1990) has argued that the procyclicality of productivity can be attributed to imperfect competition and to internal increasing returns to scale in production. In this case, productivity can be procyclical even in the absence of positive shocks to technology: a demand shock that stimulates output can be associated with increases in productivity by leading to endogenous increases in e±ciency. A third explanation, that is stressed by Abbot, Griliches, and Hausman (1988) and by Basu (1995), appeals to cyclical variations in factor utilization rates.

These alternative explanations for the procyclicality of productivity are relevant in the present context because they a®ect how one quanti¯es technology shocks. If there are constant returns to scale, all factors are fully variable, and there is perfect competition in the product and factor markets, then the conventional Solow residual described in Section 2 and tested for nonlinearity in Section 4 provides an appropriate observable measure of technology shocks. If, on the other hand, ¯rms have substantial market power due to imperfect competition, as argued by Hall (1988, 1990), then nonlinearity in the generating mechanism for the markup of price over marginal cost might spuriously cause the generating mechanism of the conventional Solow residual to appear to be nonlinear; in any case, the conventional Solow residual is no longer a valid measure of technology shocks in this instance. Similarly, if there is cyclical variation in factor utilization rates, then the conventional Solow residual inappropriately includes a component due to unobserved variation in capital and/or labor utilization rates. We allow for the possibility that product markets may not be perfectly competitive by also implementing our nonlinearity tests using a cost-based Solow residual due to Hall (1990). To control for variation in capital utilization rates, we also implement our nonlinearity tests under the assumption that capital services are proportional to electricity usage. We allow for the e®ects of variable labor utilization rates (due to labor hoarding and other factors) by examining the behavior of average hours per worker for potential nonlinearities. These results are discussed in Section 5.

The results on both the conventional and the modi¯ed Solow residuals are integrated with our test results on the input factor series (labor and capital) in Section 6. There we are able to conclude that the source of the widely-observed nonlinearities in the generating mechanism for real output is most likely in the labor markets rather than in exogenous technology shocks.

2 The Conventional Solow Residual Framework

Our benchmark model is based on the original growth accounting approach proposed by Solow (1957). Specifically, it assumes the existence of an aggregate production function that expresses aggregate output y_t as a function of capital services S_t, total hours worked L_t, and a measure of exogenous technological change z_t:

$$y_t = z_t F(S_t; L_t): \tag{2.1}$$

As is typical of this approach, we assume that capital services are proportional to the stock of capital K_t:

$$S_t = \theta K_t: \tag{2.2}$$

To derive an observable measure of exogenous technological change, Solow (1957) assumed that the function F displays constant returns to scale and that there is perfect competition in product and factor markets. Letting p_t denote the product price and w_t the wage rate, these assumptions imply that the growth rate of real output can be expressed:

$$\Delta \ln(y_t) = \alpha_t \Delta \ln(L_t) + (1 - \alpha_t) \Delta \ln(K_t) + \Delta \ln(z_t); \tag{2.3}$$

where α_t is the factor share earned by labor (the ratio of compensation $w_t L_t$ to total revenue $p_t y_t$) and where we have substituted for S_t using (2.2). Using (2.3), the Solow residual can be expressed as the difference between the growth rate of real output and the share-weighted growth rates of the inputs:

$$\Delta \ln(z_t^1) = \Delta \ln(y_t) - \alpha_t \Delta \ln(L_t) - (1 - \alpha_t) \Delta \ln(K_t): \tag{2.4}$$

The variable z_t is indexed by '1' to denote the Solow residual for our benchmark model; alternative measures of exogenous technological change which allow for imperfect competition and for variable rates of factor utilization across the business cycle are analyzed in Section 5 below.

3 Testing for Nonlinearities

In this section, we provide a brief description of the statistical tests implemented below. These include a test for ARCH effects due to McLeod and Li (1983), the BDS test proposed by Brock, Dechert, and Scheinkman (1987), and the bicovariance test due to Hinich (1995) and Hinich and Patterson (1995). These tests all share the same premise: once any linear serial dependence is removed from the data via a prewhitening model, any remaining serial dependence must be due to a nonlinear generating mechanism. Thus, each of the three procedures is actually a test of serial independence applied to the (by construction) serially uncorrelated fitting errors of an AR(p) model for the sample data. This fitting error series, standardized to zero mean and unit variance, is denoted by $\{x_t\}$ below.

<u>The McLeod-Li Test</u>

This test for ARCH effects was proposed by McLeod and Li (1983) based on a suggestion in Granger and Andersen (1978). It looks at the autocorrelation function of the squares of the prewhitened data and tests whether $\mathrm{corr}(x_t^2; x_{t-k}^2)$ is non-zero for some k. The autocorrelation function for the squared residuals $\{x_t^2\}$ is estimated by:

$$\hat{\rho}_{xx}(k) = \sum_{t=k+1}^{T} (x_t^2 - \hat{\sigma}^2)(x_{t-k}^2 - \hat{\sigma}^2) \Big/ \sum_{t=1}^{T} (x_t^2 - \hat{\sigma}^2)^2; \tag{3.1}$$

where

$$\hat{\sigma}^2 = \sum_{t=1}^{T} x_t^2 / T:$$

Under the null hypothesis that $\{x_t\}$ is an i.i.d process (and assuming that $E\left(x_t^2\right)$ exists) McLeod and Li (1983) show that, for fixed M:

$$\sqrt{T}\,\hat{\rho}_{xx} \equiv (\hat{\rho}_{xx}(1); \ldots; \hat{\rho}_{xx}(M)) \tag{3.2}$$

is asymptotically a multivariate unit normal. Thus,

$$Q^{*}_{xx} = T(T+2) \sum_{i=1}^{M} \hat{\rho}^2_{xx}(i)/(T-i); \tag{3.3}$$

is asymptotically $\chi^2(M)$ under the null hypothesis of a linear generating mechanism for the data.

The BDS Test

The BDS test is a nonparametric test for serial independence based on the correlation integral of the scalar series, $\{x_t\}$. For embedding dimension m, let $\{x_t^m\}$ denote the sequence of m-histories generated by $\{x_t\}$:

$$x_t^m \equiv (x_t, \ldots, x_{t+m-1}).$$

Then the correlation integral $C_{m,T}(\epsilon)$ for a realization of $\{x_t\}$ of length T is given by:

$$C_{m,T}(\epsilon) = \sum_{t<s} I_\epsilon(x_t^m, x_s^m) \times [2/(T_m(T_m - 1))], \tag{3.4}$$

where $T_m = T - (m-1)$ and $I_\epsilon(x_t^m, x_s^m)$ is an indicator function which equals one if the sup norm $\|x_t^m - x_s^m\| < \epsilon$ and equals 0 otherwise. Brock, Dechert, and Scheinkman (1987) exploit the asymptotic normality of $C_{m,T}(\epsilon)$ under the null hypothesis that $\{x_t\}$ is serially i.i.d. to obtain a test statistic which asymptotically converges in distribution to a unit normal.

The Hinich Bicovariance Test

This test assumes that $\{x_t\}$ is a realization from a third-order stationary stochastic process and tests for serial independence using the sample bicovariances of the data. The (r, s) sample bicovariance is defined as:

$$C_{Z3}(r, s) = (T - s)^{-1} \sum_{t=1}^{T-s} x_t x_{t+r} x_{t+s} \quad \text{for } 0 \leq r \leq s. \tag{3.5}$$

Under the null hypothesis that $\{x_t\}$ is an i.i.d. process, Hinich and Patterson (1995) show that, for $\ell < T^{.5}$,

$$CH_3 = (T - s)^{.5} \sum_{s=2}^{\ell} \sum_{r=1}^{s-1} C_{Z3}^2(r, s); \tag{3.6}$$

is asymptotically distributed chi-square with $df = (\ell - 1)\ell/2$ degrees of freedom. Hinich and Patterson (1995) recommend using $\ell = T^{.4}$ since they find that the power of the test declines for smaller values of ℓ.

4 Results Using the Conventional Solow Residual Framework

The growth rate of the conventional Solow residual ($\{\Delta \ln(z_t^1)\}$) is computed from equation (2.4) using 163 quarterly observations from 1953:I to 1993:III over which appropriately transformed measures of aggregate real output, total hours worked, and capital services appear to be stationary.

Real output is measured using real U.S. GNP; as listed in Table 1, its growth rate is denoted LY below. Two alternative measures of total hours worked are used: the ¯rst measure is manhours employed per week for all workers in all industries; the second is total employee-hours in nonagricultural establishments. As listed in Table 1, the growth rates in these two series are denoted LH1 and LH2 below; these lead to the construction of two di®erent Solow residual growth rate series, denoted SOL1 and SOL2 below, respectively. The quarterly capital stock series utilized below is constructed using computations similar to those in Christiano (1988) and Burnside, Eichenbaum, and Rebelo (1995a). Its growth rate (LC) is used in equation (2.4) but the nonlinearity tests are applied to the change in LC (denoted LCDIF below) since the time series behavior of LC itself is dominated by a unit root. These acronyms and de¯nitions are summarized in Table 1; a more detailed description of our data sources and methodology can be found in the Appendix; a time plot of the observable series is given in Figure 1 while the associated Solow residuals are plotted in Figure 2.

As noted in Section 3, all three statistical tests are implemented on prewhitened data. Each series is prewhitened using an AR(p) model, with the order p chosen to minimize the Schwartz (SC) criterion.[4] Since the sample is not very large, we do not accept these choices mechanically: we routinely check the nonlinearity test results with alternative AR(p) order speci¯cations whenever the SC-based model estimates are not clearly satisfactory, so as to verify that the test results do not materially depend on the choice made.

The nonlinearity test results for the two hours-worked series (LH1 and LH2), the real output series (LY), the capital stock series (LCDIF), and the resulting two Solow residual series (SOL1 and SOL2) are summarized in Table 2. Each entry in this Table is the marginal signi¯cance level at which the null hypothesis of a linear generating mechanism can be rejected, based on 1000 bootstrap replications.

The ¯rst thing to notice in these results is that the null hypothesis of linearity cannot be rejected for either speci¯cation of the Solow residual using any of the tests.

The second thing to notice is that the BDS and Hinich bicovariance test results con¯rm the

[4] In contrast to alternative choices (e.g., AIC or FPE), the Schwartz criterion is known to be consistent for AR(p) order determination under the null hypothesis of a linear generating mechanism; see Judge, et al. (1985, p. 246).

results from the previous studies cited in Section 1: the null hypothesis of a linear generating mechanism for aggregate real output can be rejected at the 1-2% level of significance.

Finally, note that the null hypothesis of linearity cannot be rejected for the capital stock series (LCDIF) but can be rejected at the 2-5% level for one of the hours worked series (LH1) and can be resoundingly rejected for the other, LH2. Evidently, the nonlinearity in the generating mechanism for aggregate real output is arising in the labor markets.

5 Extending These Results to More General Measures of Exogenous Technological Change

The conventional approach to deriving Solow residuals has been criticized on a number of grounds, including the fact that it does not take into account variable rates of factor utilization across the business cycle or the existence of imperfect competition in product or factor markets. In this Section, we describe the results of implementing our nonlinearity tests on alternative measures of technology shocks that take into account some of these criticisms.

The first alternative that we consider relaxes the assumption that capital services are proportional to the stock of capital. In his original paper, Solow (1957) allowed for the possibility that capital utilization rates could vary across the business cycle by measuring capital services as the product of the physical stock of capital and the employment rate. Other approaches to adjusting for variable capital utilization include measures of electricity usage (Jorgenson and Griliches 1967), the Federal Reserve Board capacity utilization series (Tatom 1980), and shift data (Shapiro 1986 and Mayshar and Solon 1993). Recently, Burnside, Eichenbaum, and Rebelo (1995a) have argued that industrial electricity consumption may serve as a reasonable proxy for capital services. As a simple alternative that allows for variable capital utilization, we follow Burnside, Eichenbaum, and Rebelo (1995a,b) and assume that aggregate electricity usage, E_t, is proportional to capital services:[5]

$$E_t = \phi S_t. \tag{5.1}$$

[5] Costello (1993) also measures capital services by electricity consumption in her study of productivity across countries.

However, our definition of the electricity usage series differs from Burnside, Eichenbaum, and Rebelo (1995a). The series that we use on electricity usage is defined as the monthly index of electric utility sales to commercial and other users whereas Burnside et al use is a monthly index of total electrical power usage in the industrial sector (manufacturing plus mining plus utility industries). Using the relationship (5.1) in the aggregate production function and proceeding as before yields an alternative expression for the Solow residual as:

$$\Delta\ln(z_t^2) = \Delta\ln(y_t) - \alpha_t\Delta\ln(L_t) - (1-\alpha_t)\Delta\ln(E_t). \qquad (5.2)$$

The growth rate in aggregate electricity usage is denoted LECTRIC and plotted in Figure 1 below.[6] Equation (5.2) then yields two alternative Solow residual series, denoted SOLE1 and SOLE2 respectively, depending on which of the two hours worked series is used. SOLE1 and SOLE2 are plotted in Figure 2. The nonlinearity test results on LECTRIC, SOLE1, and SOLE2 are summarized in Table 3. None of these results would allow one to reject the null hypothesis of a linear generating mechanism for these time series at the 5% level.

Another explanation that has been suggested for the observed behavior of Solow residuals is given by labor hoarding, also known as reserve labor or variable utilization of labor. For example, Rotemberg and Summers (1990) present a model with inflexible prices and labor hoarding which generates the procyclical movements in productivity observed in the data. In this vein, Burnside, Eichenbaum, and Rebelo (1993) modify Hansen's (1985) real business cycle model with indivisibilities to allow for government expenditure shocks and labor hoarding. In their model, total hours worked depends on the number of workers employed times their effective work effort. Letting N_t denote the number of workers who are employed and W_t the level of effort expended by an individual, output is assumed to be produced according to the Cobb-Douglas production function:

$$y_t = z_t K_t^{1-\alpha}[f N_t W_t]^{\alpha}, \qquad (5.3)$$

where fW_t denotes total effective work and $0 < \alpha < 1$. Proceeding as before, the Solow residual is:

$$\Delta\ln(z_t^4) = \Delta\ln(y_t) - \alpha[\Delta\ln(N_t) + \ln(W_t)] - (1-\alpha)\Delta\ln(K_t). \qquad (5.4)$$

[6] The statistical behavior of this time series was notably affected by a pair of outliers in 1973:IV and 1974:I; consequently, these two observations were set equal to the sample mean.

This expression shows that unmeasured variation in work e®ort enters as an additional determinant of observed measures of productivity.[7] In the absence of variation in work e®ort, total hours worked L_t equals the total number of individuals at work, N_t, times the ¯xed shift length, f:

$$L_t = N_t f: \tag{5.5}$$

Under this assumption, the conventional Solow residual is related to the Solow residual derived from the labor hoarding model as follows:

$$¢ \ln(z_t^1) = ¢ \ln(z_t^4) + ® ¢ \ln(W_t): \tag{5.6}$$

If z_t^4 is taken to be identical to the \true" technology shock z_t, then the expression in (5.6) implies that the conventional Solow residual can confound movements in technology with movements in work e®ort across the cycle, which itself responds to exogenous \demand shocks," such as government consumption shocks.

Following the approach in Abbott, Griliches, and Hausman (1988) or Caballero and Lyons (1992), we allow for the e®ects of variable labor (and capital) utilization on real output growth and the Solow residual by testing the behavior of average hours worked per worker for potential nonlinearities. The growth rate in average hours worked is denoted LHAVG and plotted in Figure 1. The nonlinearity test results for LHAVG are summarized in Table 3; the generating mechanism for this time series appears to be linear. Thus, our ¯nding that the generating mechanism for the conventional Solow residual is linear is consistent with the existence of movements in W_t, but we cannot rule out the possibility that variation in W_t is obscuring the existence of nonlinear behavior in f$¢ \ln(z_t^4)$g.

In a series of papers, Hall (1988, 1990) showed that the derivation of the Solow residual could be extended to situations in which there may exist market power by ¯rms and increasing returns to scale in production. The ¯nal variant of our framework follows Hall's approach. Let ° denote the returns to scale of the aggregate production function. Suppose also that ¯rms are not necessarily price-takers in the product market and hence do not equate the price of their output to marginal

[7] In Burnside, Eichenbaum, and Rebelo's (1993) framework, labor hoarding emerges because the level of employment N_t is chosen before observing current realizations of the true technology shock and the government expenditure shock. Other recent analyses of labor hoarding include Horning (1994) and Fairise and Langot (1994).

cost. Letting r_t denote the service price of capital and defining α_t^c as the share of labor in total costs, $\alpha_t^c \equiv w_t L_t/(w_t L_t + r_t K_t)$, the solution to the cost-minimization problem of a firm that takes factor prices as given implies that the the growth rate of real output must equal γ times the cost-share weighted growth in inputs plus productivity growth:

$$\Delta \ln(y_t) = \gamma\,[\alpha_t^c\,\Delta \ln(L_t) + (1 - \alpha_t^c)\,\Delta \ln(K_t)] + \Delta \ln(z_t). \tag{5.7}$$

As Caballero and Lyons (1992) note, this derivation is valid under the assumption that firms' optimization problems can be approximated by a sequence of static problems. Note that the implied Solow residual from equation (5.7) – the "cost-based Solow residual" – requires data on the rental price of capital, r_t.

The cost-based Solow residual also depends on the unknown parameter γ. The evidence on the degree of returns to scale is mixed. Burnside, Eichenbaum, and Rebelo (1995) show that constant returns to scale at the aggregate level cannot be rejected when they proxy capital services by electricity usage. And Basu and Fernald (1995, 1996) find that controlling for aggregation bias under imperfect competition leads to estimates of γ that are not significantly different from one. Consequently, γ is set to one here, yielding:

$$\Delta \ln(z_t^*) = \Delta \ln(y_t) - [\alpha_t^c\,\Delta \ln(L_t) + (1 - \alpha_t^c)\,\Delta \ln(K_t)]. \tag{5.8}$$

for the cost-based Solow residual.

Four cost-based Solow residuals are computed using equation (5.8) – denoted SOLC1, SOLC2, SOLCE1, and SOLCE2 – depending on which of the two hours worked series (LH1 or LH2) and which of the two capital stock series (LCDIF or LECTRIC) is used. Their plots are in Figure 2. The nonlinearity test results for these four alternative Solow residual specifications are summarized in Table 4; there is no evidence of nonlinearity in the generating mechanisms for any of them.[8]

The results in this Section might seem negative, but they are actually quite significant. They demonstrate that the results described in Section 4 using the conventional specification are robust with respect to all of these alternative specifications for the Solow residual.

[8] The results for SOLC2 are based on observations up to 1987:4 because the series appears to exhibit nonstationarity over the full sample.

6 Conclusion

We have presented the results of several alternative tests for nonlinearity in the generating mechanisms of real GNP, the inputs to an aggregate production function, and the Solow residuals derived under several sets of assumptions about the measurement of inputs and the nature of competition in product markets. We find substantial evidence that the generating mechanism for real GNP is nonlinear, but no evidence at all for nonlinearity in the generating mechanism for the Solow residuals under any of the different specifications that we studied. In principle, this result for the Solow residuals could be due to insufficient power in our tests due to the small size of the sample. However, the fact that we do detect nonlinearity in the generating mechanisms for real GNP growth and for the growth rate of total hours worked over the same sample period indicates that the power of the test is not the problem: we are not detecting nonlinearity in the Solow residuals because there simply isn't much there to detect.

This result implies that it is the macroeconomy itself which is nonlinear – not the technology (or factor productivity) shocks that are impinging on, and in part, driving it. Thus, nonlinear models for the behavior of aggregate output, which allow for the asymmetric response of aggregate output to exogenous shocks over the business cycle need to be considered rather than nonlinear models for the shocks themselves. And – since these results clearly indicate that any statistically adequate macroeconomic model must be significantly nonlinear – the modelling of rational expectations formation must explicitly take this nonlinearity into account.

Since the generating mechanisms for the measures of the capital services input do not appear to be significantly nonlinear, our results strongly suggest that the nonlinearity in real output documented in this and previous studies can be largely attributed to the nonlinearity we observe in the generating mechanism for hours worked. Furthermore, the fact that we do not find nonlinearities in the generating mechanism for the average hours worked per worker variable indicates that the nonlinearity in the hours worked variable arises in the generating mechanism for the number of employees rather than in the generating mechanism for work effort. This observation is broadly consistent with the models of labor hoarding discussed above; it suggests that a fruitful direction of research would be to analyze alternative models of labor hoarding to determine whether they are capable of generating the kinds of nonlinear behavior implied by our results.

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Table 1

Variable Names

LY: growth rate of real GNP
LH1: growth rate of hours worked for all workers, all industries
LH2: growth rate of employee-hours in nonagricultural establishments
LC: growth rate of physical capital stock
LCDIF: di®erences of growth rate of physical capital stock
LECTRIC: growth rate of electricity usage
LHAVG: growth rate of average hours per worker
SOL1: LY- SHARE*LH1 -(1-SHARE)*LC
SOL2: LY- SHARE*LH2 -(1-SHARE)*LC
SOLE1: LY- SHARE*LH1 -(1-SHARE)*LECTRIC
SOLE2: LY- SHARE*LH2 -(1-SHARE)*LECTRIC
SOLC1: LY- SHAREC*LH1 -(1-SHAREC)*LC
SOLC2: LY- SHAREC*LH2 -(1-SHAREC)*LC
SOLCE1: LY- SHAREC*LH1 -(1-SHAREC)*LECTRIC
SOLCE2: LY- SHAREC*LH2 -(1-SHAREC)*LECTRIC

Table 2

Test Resultsy

	LY	LH1	LH2	LCDIF	SOL1	SOL2
Mc-Leod-Li Test						
M = 1	0:407	0:939	0:172	0:601	0:704	0:466
M = 2	0:229	0:718	0:047¤	0:868	0:869	0:397
M = 3	0:344	0:667	0:038¤	0:829	0:243	0:380
M = 4	0:416	0:541	0:014¤	0:918	0:306	0:543
M = 8	0:191	0:526	0:015¤	0:380	0:650	0:867
BDS Test						
² = 0:5						
m = 2	0:359	0:582	0:002¤¤	0:531	0:434	0:154
m = 3	0:221	0:285	0:001¤¤	0:345	0:441	0:091
m = 4	0:365	0:059	0:002¤¤	0:395	0:712	0:123
² = 1:0						
m = 2	0:363	0:163	0:006¤¤	0:635	0:876	0:243
m = 3	0:037¤	0:080	0:000¤¤	0:755	0:809	0:154
m = 4	0:023¤	0:053	0:000¤¤	0:775	0:878	0:133
² = 2:0						
m = 2	0:203	0:186	0:092	0:378	0:607	0:119
m = 3	0:040¤	0:095	0:019¤	0:511	0:566	0:079
m = 4	0:022¤	0:144	0:006¤¤	0:626	0:730	0:092
Hinich Bicovariance Test						
` = 7	0:012¤	0:018¤	0:000¤¤	0:605	0:404	0:088

yMarginal signi¯cance levels. All results are based on 1000 bootstrap replications. Results signi¯cant at the 5% level are marked with an `*'; results signi¯cant at the 1% level are marked with a `**'. `M' is

maximum lag used in McLeod-Li test; `m' is embedding dimension for BDS test; and ``' is maximum lag used in Hinich bicovariance test.

Table 3

Test Resultsy

	LECTRIC	SOLE1	SOLE2	LHAVG
Mc-Leod-Li Test				
M = 1	0:241	0:620	0:323	0:197
M = 2	0:438	0:660	0:613	0:217
M = 3	0:197	0:616	0:819	0:373
M = 4	0:112	0:360	0:571	0:461
M = 8	0:246	0:498	0:521	0:873
BDS Test				
² = 0:5				
m = 2	0:107	0:945	0:257	0:424
m = 3	0:294	0:940	0:107	0:654
m = 4	0:199	0:891	0:153	0:786
² = 1:0				
m = 2	0:125	0:558	0:285	0:340
m = 3	0:149	0:622	0:159	0:281
m = 4	0:242	0:709	0:117	0:231
² = 2:0				
m = 2	0:170	0:270	0:125	0:191
m = 3	0:231	0:320	0:077	0:224
m = 4	0:380	0:426	0:057	0:121
Hinich Bicovariance Test				
` = 7	0:808	0:342	0:147	0:028¤

yMarginal signi¯cance levels. All results are based on 1000 bootstrap replications. Results signi¯cant at the 4% level are marked with an `*'; results signi¯cant at the 1% level are marked with a `**'. M' is

maximum lag used in McLeod-Li test; `m' is embedding dimension for BDS test; and ``' is maximum lag used in Hinich bicovariance test.

Table 4

Test Resultsy

	SOLC1	SOLC2z	SOLCE1	SOLCE2
Mc-Leod-Li Test				
M = 1	0:709	0:981	0:461	0:195
M = 2	0:915	0:771	0:707	0:350
M = 3	0:304	0:893	0:689	0:441
M = 4	0:417	0:710	0:332	0:492
M = 8	0:621	0:902	0:461	0:682
BDS Test				
^{2} = 0:5				
m = 2	0:360	0:859	0:840	0:174
m = 3	0:610	0:831	0:914	0:130
m = 4	0:646	0:893	0:975	0:056
^{2} = 1:0				
m = 2	0:743	0:742	0:381	0:264
m = 3	0:826	0:569	0:543	0:304
m = 4	0:795	0:428	0:546	0:184
^{2} = 2:0				
m = 2	0:488	0:597	0:229	0:074
m = 3	0:571	0:448	0:387	0:071
m = 4	0:643	0:295	0:518	0:057
Hinich Bicovariance Test				
` = 7	0:362	0:065	0:354	0:086

yMarginal signi¯cance levels. All results are based on 1000 bootstrap replications. Results signi¯cant at the 5% level are marked with an `*'; results signi¯cant at the 1% level are marked with a `**'. M' is

maximum lag used in McLeod-Li test; `m' is embedding dimension for BDS test; and ``' is maximum lag used in Hinich bicovariance test.

Data

The data are quarterly data for the aggregate economy. Real output is measured as gross national product in 1987 dollars from the National Income and Product Accounts (NIPA), Table 1.10. We measured total hours worked in two different ways: first, as manhours employed per week for all workers, all industries, derived from the Household Survey of the Bureau of Labor Statistics publication, The Employment Situation, and second, as total employee-hours for wage and salary workers in nonagricultural establishments. The corresponding CITIBASE codes are LHOURS and LPMHU, respectively. Multiplying the first variable LHOURS, namely, manhours worked per week for all workers, all industries by the number of weeks in a quarter, yields our first measure of total hours worked. Our second measure of total hours is defined directly from the variable LPMHU. We measured average hours worked by dividing the monthly series LPMHU by the monthly series LPNAG, which denotes nonfarm employment. The series on electricity usage is defined as the monthly index of electric utility sales to commercial and other users; its CITIBASE code is IPCOE. All quarterly series are derived as three month averages of the monthly series.

There is no published quarterly data on different components of the aggregate capital stock. We obtained annual data from the Bureau of Economic Analysis capital stock tables described in the publication, Fixed Reproducible Tangible Wealth of the U.S., 1989. These data are for the period 1946-1993 and include annual measures of the gross and net stocks of private nonresidential structures and producers' durable equipment (which comprise the stock of fixed nonresidential private capital), residential capital, and government owned fixed capital consisting of equipment and structures in 1987 dollars. Our measure of the aggregate net capital stock is obtained as the sum of the different components of the gross capital stocks, interpolated to a quarterly basis using the method in Fernandez (1981), and corrected for depreciation. We used quarterly data on gross investment in nonresidential structures, producers' durable equipment, and residential structures from the NIPA Table 5.5 to construct the corresponding components of the gross capital stocks. Likewise, quarterly data on the consumption of fixed capital, NIPA Table 1.10, and the rental income of persons with capital consumption adjustment, NIPA Table 1.14, were used to derive quarterly measures of depreciation for the fixed private nonresidential and residential capital stocks, respectively. Finally, quarterly series of the net stock of government owned fixed capital was linearly interpolated from the annual measure using the quarterly stock of fixed private nonresidential capital.[9]

The share of labor in national income denoted α_t is constructed as the ratio of total employee compensation to national income, NIPA Table 1.14. To calculate the labor share in costs denoted α_t^c, an estimate of the rental rate of capital is required. Following Hall and Jorgenson (1967), this is calculated as:

$$r_t = (\delta + v_t)\frac{1 - z_t\tau_t - \omega_t}{1 - \tau_t}p_{kt};$$

where δ is the average depreciation rate, v_t is the required rate of return on capital (measured as the dividend yield on the Standard and Poor 500 portfolio), z_t is the present discounted value of depreciation allowances, ω_t is the investment tax credit rate, τ_t is the profits tax rate, and p_{kt} is the deflator for business fixed investment, NIPA Table 7.1. The value of δ was taken to be 0.021. We obtained unpublished data on the present discounted value of depreciation allowances z_t, the investment tax credit ω_t and current value of the capital stocks of corporate and noncorporate capital from Dale Jorgenson. We constructed an aggregate cost of capital variable by weighting the cost of capital for each sector by the current value of the stocks of corporate and noncorporate capital. The average marginal tax rates used to construct the cost of capital variables are from Jorgenson and Yun (1995).

The calculation of the Solow residuals depends on the particular specification that is used. For example, the Solow residual for the benchmark model is computed as $\Delta\ln(z_t^1)$ from equation (2.4). We replace α_t by $\bar{\alpha}_t = .5(\alpha_t + \alpha_{t-1})$ in all the relevant expressions to obtain a Tornquist index of multi-factor productivity. We omitted observations on all the series prior to 1953 to obtain a sample of 163 observations, from 1953:I to 1993:III.

[9] Our constructed measure of the physical capital is similar to that used by Christiano (1988) and Burnside, Eichenbaum, and Rebelo (1995) except for the fact that it excludes the stock of consumer durables.