GROWTH MAY BE GOOD FOR THE POOR, BUT DECLINE IS DISASTROUS: ON THE NON-ROBUSTNESS OF THE DOLLAR-KRAAY RESULT

Richard Ashley*
Virginia Tech

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abstract

The Dollar-Kraay result (that the income elasticity of the lowest quintile's income is essentially one) is identified as a statistical artifact related to the irregular sampling intervals in their data. Corrected results suggest an asymmetric response to growth versus decline.

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*Economics Department (0316), Virginia Tech, Blacksburg VA 24061. Telephone: (540) 231 6220; fax: (540) 231 5097; e-mail: ashleyr@vt.edu. The author gratefully acknowledges helpful comments from Dani Rodrik and a number of anonymous referees.

1. Introduction

The proposition that "growth is good for the poor," in the sense that the real per capita income of the poor in a country is directly related to average real per capita income, is not controversial. However, Dollar and Kraay (2002) – "DK" below – further claim that the elasticity of the income of the poor relative to mean income is statistically indistinguishable from unity. Thus, they assert that recent international economic growth has been equiproportional – i.e., income distribution neutral.

The definition and construction of DK's dataset and their focus on this particular elasticity have been criticized by others – e.g., Weisbrot, et al. (2001), Lübker, et al. (2002), and Bourguignon (2003). Weisbrot, et al. (2001, Appendix B) emphasize a number of measurement error problems in the DK data set. Lübker, et al. (2002) present a number of methodological criticisms, including the lack of a coherent theory underlying the DK estimation model, the way that DK adjust their data for cross-country definitional variations, and the aggregation of the data set across income levels. And Bourguignon (2003) emphasizes the inherent relationships between growth and changes in the distribution of income, which are glossed over in the DK approach. Here, instead, the DK data set and basic framework are taken as given and the question addressed: is the DK unit-elasticity conclusion in fact supported by their data?

2. Summary of the DK Study

DK model the relationship between the per capita real income of the poor and overall real per capita income using:

$$Y^p_{c,t} = \alpha_o + \alpha_1 Y_{c,t} + \alpha_2 X_{c,t} + \mu_c + \eta_{c,t} \tag{1}$$

where $Y_{c,t}$ is the logarithm of real per capita income of country c in year t, X_{ct} is a vector of control variables, and $Y^p_{c,t}$ is the logarithm of income accruing to the poorest 20% of the population of country c in year t. DK use $y_{c,t}$ and $y^p_{c,t}$ for these two income measures; uppercase symbols are used here so that the corresponding lowercase symbols can be used below to denote the corresponding growth rates.

DK also consider this model in growth rate form:

$$y^p_{c,t} = \beta_1 y_{c,t} + \beta_2 x_{c,t} + \epsilon_{c,t} \tag{2}$$

where each lowercase variable in this regression equation is the change (over its previous observation) in the corresponding uppercase variable from the previous equation.

DK's data set is irregularly sampled, thus:

$$y_{c,t} \equiv Y_{c,t} - Y_{c,t-k(c,t)} \tag{3}$$

where k(c,t) is the number of years elapsed since the immediately prior observation in the data set. In fact – as turns out to be important below – DK's data set is **quite** irregularly spaced: k(c,t) ranges from 5 to 37 years. Table 1 below tabulates the distribution of k(c,t) across the DK sample.

Table 1
Distribution of Inter-Observation Intervals in the DK Data Set

k(c,t) {in years}	Number of observations
5	135
6 -7	83
8 - 15	54
16 - 22	9
23 - 37	4

DK estimate Equations 1 and 2 simultaneously as a system, using the Arellano and Bover (1995) dynamic panel regression estimator, imposing the constraints that α_1 equals β_1 and that α_2 equals β_2. They find that α_1 and β_1 are not statistically distinguishable from one and that this result is robust to inclusion of a variety of control variables listed in their paper.

3. Critique of DK's Econometric Methodology

Imposing their parameter restrictions and (for simplicity of exposition) suppressing the control variables, the DK model is:

$$Y_{c,t}^{p} = \alpha_o + \alpha_1 Y_{c,t} + \mu_c + \eta_{c,t} \tag{4}$$

$$y_{c,t}^{p} = \alpha_1 y_{c,t} + \epsilon_{c,t} = \alpha_1 y_{c,t} + \left\{\eta_{c,t} - \eta_{c,t-k(c,t)}\right\} \tag{5}$$

where ϵ_{ct} is the difference between successive values of η_{ct}, and the two instrument equations, for use in the Arellano and Bover (1995) estimator, are:

$$Y_{c,t} = \gamma_o + \gamma_1 \tilde{y}_{c,t} + \nu_{c,t} \tag{6}$$

$$y_{c,t} = \lambda_1 Y_{c,t-k(c,t)} + \lambda_2 \tilde{y}_{c,t-k(c,t)} + \zeta_{c,t} \tag{7}$$

where $\tilde{y}_{c,t}$ is the growth in mean income over the five years preceding year t.

Two inter-related features of this formulation are problematic. First, note that – in view of the fact that the inter-observational interval k(c,t) varies from 5 to 37 years – the coefficients λ_1 and λ_2 in Equation 7, the first-stage instrument regression for $y_{c,t}$, do not have unique, well-defined population values. For example, since λ_1 is $\partial E\{y_{c,t}\}/\partial Y_{c,t-k(c,t)}$, its value will differ for each value of k(c,t). Thus, neither λ_1 nor λ_1 can be consistently estimated using these data; consequently, the second-stage estimates cannot be consistent either.

Second, note that

$$\epsilon_{c,t} = \eta_{c,t} - \eta_{c,t-k(c,t)} \tag{8}$$

implies that at least one of these two error terms – $\epsilon_{c,t}$ or $\eta_{c,t}$ – must be highly autocorrelated. Results in the next section, on an equi-spaced subset of the DK data set, indicate that $\eta_{c,t}$ is the likely offender. This result strongly suggests a need for inclusion of first-order dynamics in the levels equation, which DK do not (indeed, cannot) include, again because one "lag" in their data set ranges from 5 years to 37 years in length.[1]

Because the instrument for the growth rate equation seems fatally flawed and at least one of the two DK equations suffers from a serious problem with unmodeled dynamics, DK's conclusion that the null hypotheses $\alpha_1 = 1$ and $\beta_1 = 1$ cannot be rejected is of very doubtful validity.

[1]A referee for an early version of this paper claims that DK's intent was to assume first-order serial correlation from one irregularly-spaced observation to the next. In view of the wildly varying inter-observational intervals in the DK data set, this expedient seems rather an heroic assumption.

4. Meaningful Estimates from the DK Data Set

One approach for obtaining meaningful estimates from the DK data set is to simply drop all of the observations with k(c,t,) greater than five. That yields 135 equi-spaced observations, but this sample truncation would likely exacerbate the existing sample selection biases caused by the fact that the data set is already unbalanced. Since the median number of growth rate observations per country is only three in the full data set, it is better to recognize that this was never much of a panel to begin with and reduce it to a cross-section by using only the most recent observations on each country.

This approach is implemented here, yielding a cross-section regression over 92 countries. This is considerably less data than the 285 observations DK employ, but the later data are arguably more relevant to policy and to the broader question of the implications of globalization-induced growth on income inequality. In any case, the earlier data are of lesser utility because they are sampled at such irregular time intervals.[2] It is also easier to interpret the results from a data set which does not artificially emphasize the countries for which larger amounts of data are available. Of course, the worry with this smaller sample is that the parameter estimates may be insufficiently precise as to yield useful results; fortunately, that is not the case here.

2SLS estimation of this model yields:

$$y^{p}_{c,T_c} = \underset{(.44)}{-1.43} + \underset{(.22)}{1.61}\; y_{c,T_c} + e_c^{\{1\}} \qquad R^2 = .528 \qquad (9)$$

[2]And it also doesn't make much difference: very similar results are quoted in a robustness check at the end of this section using a cross-section of growth rates averaged over the entire sample period available for each country.

where T_c is the date of the last observation on country c and where the instruments used are agrprodav, eap, eca, landav, and rulelaw, yielding a first-stage $\bar{R}^2$ value of .375. These variables are defined in DK (2002) and in Appendix 1 below; the first-stage regression results are given in Appendix 2. Since lagged variables are not used as instruments, the first-stage equation coefficients have well-defined values here, even though $k(c,T_c)$ and T_c are not completely homogeneous across the sample. The fitting errors, $e_c^{\{1\}}$, appear to be gaussian based on the Shapiro-Wilk and skewness-kurtosis tests. Robust (White) standard error estimates are quoted, here and below.

Both y^p_{c,T_c} and y_{c,T_c} are annualized growth rates over a period of length $k(c, T_c)$ ending in year T_c, the last available observation for country c. Note that annualization departs from DK's definition (Equation 3), dividing by $k(c, T_c)$. This ensures that each country's data are weighted equally, regardless of the inter-observational time interval; it also reduces the likely degree of heteroscedasticity in the equation error term, increasing the efficiency of the parameter estimates. T_c is not identical for all countries, but $T_c \geq 1987$ for 86 of the 89 countries for which data are available on all of the variables used in the instruments.

The coefficient of 1.61 on y_{c,T_c} in Equation 9 is significantly different from one at the 1% level, but this result is invalid in view of the fact that the underlying coefficient on y_{c,T_c} varies substantially across the sample. In particular, the following estimated model demonstrates that the value of this coefficient for countries which are growing $(y_{c,T_c} \geq 0)$ differs notably from that for countries which are declining – i.e., experiencing negative real per capita growth, $(y_{c,T_c} < 0)$:

$$\underset{}{y^p_{c,T_c}} = \underset{(.94)}{-1.46} + \underset{(.33)}{0.44}\, y^+_{c,T_c} + \underset{(.33)}{2.52}\, y^-_{c,T_c} + e_c^{\{2\}} \qquad R^2 = .588 \tag{10}$$

where y^{+}_{c,T_c} is equal to y_{c,T_c} for each country with $y_{c,T_c} \geq 0$ and otherwise zero and y^{-}_{c,T_c} is analogously defined for the countries with $y_{c,T_c} < 0$. Based on this estimated model, the null hypothesis that the coefficients on y^{+}_{c,T_c} and y^{-}_{c,T_c} are equal can be rejected with P = .0004; the null hypothesis that the coefficient on y^{+}_{c,T_c} equals one can be rejected with P = .09; and the null hypothesis that the coefficient on y^{-}_{c,T_c} equals one can be rejected with P <.00005.[3]

A number of variations on this model were estimated: using different instruments (or lagged instruments, or no instruments at all), including one or more of DK's control variables, and/or incorporating restrictions so as to make the growth rate interval {k(c, T_c)} or the ending-year (T_c) more homogeneous across the sample. All yield very similar results to that of Equation 10, except that the models for which the observations with larger values of k(c, T_c) have been eliminated tend to fit better; also, the coefficient on y^{+}_{c,T_c} is usually both smaller and significantly different from one at the 5% level in these models. For example, restricting the sample to the 68 observations for which k(c, T_c) $\leq$ 8 and $T_c \geq$ 1990, so that the distribution of inter-observation intervals is

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[3]See also Weisbrot, et al. (2001, p. 7), where they too direct attention to the countries with negative growth rates.

Table 2
Distribution of Inter-Observation Intervals

k(c,t) {in years}	Number of observations
5	36
6	17
7	12
8	3

yields:

$$y^{p}_{c,T_c} = 1.92 + 0.27\, y^{+}_{c,T_c} + 2.47\, y^{-}_{c,T_c} + e^{\{3\}}_{c} \qquad R^2 = .635 \qquad (11)$$
$$\quad (1.17) \quad (.36) \qquad (.32)$$

For this estimated model, the null hypothesis that the coefficients on y^{+}_{c,T_c} and y^{-}_{c,T_c} are equal can be rejected with P = .0005; the null hypothesis that the coefficient on y^{+}_{c,T_c} equals one can be rejected with P = .05; and the null hypothesis that the coefficient on y^{-}_{c,T_c} equals one can be rejected with P <.00005.

Similar results were also obtained estimating models in levels – i.e., Equation 4 – in which only the observations for which k(c, t) equals five were retained, so that lagged endogenous and explanatory variables could be included in the equation to eliminate first-order serial correlation in the fitting errors:

$$Y^{p}_{c,t} = -2.70 + 1.18\, Y^{+}_{c,t} + 1.24\, Y^{-}_{c,t} + e^{\{4\}}_{c,t} \qquad R^2 = .870 \qquad (12)$$
$$\quad (1.14) \quad (.13) \qquad (.14)$$

$$Y^{p}_{c,t} = -0.30 + .68\, Y^{p}_{c,t-5} \qquad (13)$$
$$\quad (.22) \quad (.06)$$
$$+ .73\, Y^{+}_{c,t} - .41\, Y^{+}_{c,t-5} + 1.21\, Y^{-}_{c,t} - .911\, Y^{-}_{c,t-5} + e^{\{5\}}_{c,t} \qquad R^2 = .968$$
$$\quad (.30) \quad (.29) \quad (.63) \quad (.65)$$

Here $Y_{c,t}^{+}$ equals $Y_{c,t}$ for observations in which $y_{c,t} \geq 0$ and is zero otherwise; $Y_{c,t}^{-}$ is defined analogously. The first-stage regression estimates for these two equations are given in Appendix 3. The instruments used for Equation 12 were agrprodav, eap, and eca; the lagged values $Y_{c,t-5}^{p}$, $Y_{c,t-5}^{+}$, and $Y_{c,t-5}^{-}$ were additionally included for Equation 13. Note that the lagged dependent variable $\left(Y_{c,t-5}^{p}\right)$ enters Equation 13 with a fairly large and highly significant coefficient estimate, suggesting that it is $\eta_{c,t}$ in Equation 4 rather than $\epsilon_{c,t}$ in Equation 5 which is substantially autocorrelated. Note also that Equation 12 – which omits the dynamic terms, as in DK's model – restores the DK result that the coefficient on $Y_{c,t}$ essentially equals one, both for countries with positive growth rates and for countries with negative growth rates. This result strongly suggests that DK's inability to detect this asymmetry in their model estimates was caused by their failure to include the necessary dynamics in the levels-equation portion of their estimation model, due to the highly irregular inter-observational intervals in their data set.

As noted at the outset of this section, the existing sample selection biases in the DK data set are substantially exacerbated by the restriction (in Equations 12 and 13) of the estimation to only the subset of observations for which k(c, t) equals five. Moreover, the 2SLS estimates of these levels models are at once much noisier and also much more sensitive to different instrument choices than are the analogous estimates of the growth rate models. Consequently, while the signs and sizes of the coefficient estimates in Equation 13 are much as one might expect from the growth-rate model estimates, Equation 13 is not useful for testing the relevant hypotheses.[4]

As a final robustness check, the growth rate model (Equation 10) was re-estimated using the average growth rate for each country over the entire available sample period, rather than the most recent growth rate which is feasible to compute. The resulting 2SLS estimated model, using the same instruments as in Equations 9 and 10, is:

$$\underset{}{\bar{y}_c^{\,p}} = \underset{(.69)}{0.75} + \underset{(.23)}{0.77}\,\bar{y}_c^{+} + \underset{(.32)}{2.36}\,\bar{y}_c^{-} + e_c^{\{5\}} \qquad R^2 = .708 \qquad (14)$$

where the first-stage estimates are quoted in Appendix 2. Based on this estimated model, the null hypothesis that the coefficients on $\bar{y}_c^{+}$ and $\bar{y}_c^{-}$ are equal can be rejected with P = .002; the null hypothesis that the coefficient on $\bar{y}_c^{+}$ equals one can be rejected with P = .32; and the null hypothesis that the coefficient on $\bar{y}_c^{-}$ equals one can be rejected with P <.00005. Thus, the results are essentially identical regardless of which growth rate formulation one chooses.

Therefore it seems reasonable to conclude from the DK data set that the coefficients α_1 and β_1 on overall per capita real income in DK's models (Equations 1 and 2 above) are perhaps a little less than one for countries with positive growth rates and very likely substantially in excess of one for countries with negative growth rates. It appears that the only way to produce contrary results is to estimate a model in levels and wrongly omit the relevant dynamics.

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[4]OLS estimation of Equation 13 – while no doubt less credible in terms of bias – yields more precise parameter estimates. In particular, the OLS estimated coefficients on $Y_{c,t}^{+}$ and $Y_{c,t}^{-}$ are .96 ± .14 and 1.68 ± .30, respectively, allowing one to reject at the 5% level both the null hypothesis that the coefficient on $Y_{c,t}^{-}$ is one and the null hypothesis that the coefficients on $Y_{c,t}^{+}$ and $Y_{c,t}^{-}$ are equal.

5. Conclusions

DK's principal conclusion – that the elasticity of the income of the lowest quintile with respect to mean income is statistically indistinguishable from one – is evidently not supported by their data. In fact, a re-analysis of their data weakly indicates that the coefficients α_1 and β_1 on overall per capita real income in Equations 1 and 2 are likely a bit less than one for countries which are growing and strongly indicates that these coefficients are substantially greater than one for countries which are declining.

These results suggest that the poorest quintile probably does not share proportionately in growth, but bears the brunt of any decline in real income. One might then conclude that income inequality increases either way, but more quickly for economies in decline. However, in view of the above-noted defects in the DK framework and data, it seems inappropriate to generalize based solely on results obtained using these data. Consequently, this asymmetry observation is left here as a conjecture to be tested using other data sets.

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Appendix 1
Definitions of Instruments[5]

agroprodav	Agriculture Relative Labor Productivity	Current price share of agriculture in GDP divided by share of workforce in agriculture.
eap	East Asia and Pacific Regional Dummy	
eca	Europe and Central Asia Regional Dummy	
landav	Arable Land per Worker	Total arable land in hectares divided by population aged 15-64.
rulelaw	Rule of Law	Index, greater values equal better rule of law.

[5]These variables were defined and used (as control variates) in DK (2002, p. 220); their definitions are summarized here for the reader's convenience. The original source of these data is the World Bank, except for the rule of law index, which is from a World Bank working paper by Kaufmann, et al. (1999). The DK data set is available at www.worldbank.org/research/growth.

Appendix 2
First-Stage Regression Estimates – Models in Growth Rates[a]

	Eqn. 9	Eqn. 10		Eqn. 11		Eqn. 14	
	y_{c,T_c}	y^{+}_{c,T_c}	y^{-}_{c,T_c}	y^{+}_{c,T_c}	y^{-}_{c,T_c}	$\bar{y}^{+}_{c}$	$\bar{y}^{-}_{c}$
constant	2.21 (.79)	1.61 (.48)	0.60 (.48)	1.63 (.58)	1.62 (.56)	1.79 (.37)	0.52 (.41)
agroprodav	-2.87 (.89)	-1.24 (.55)	-1.64 (.54)	-1.40 (.72)	-3.28 (.70)	-1.00 (.41)	-1.37 (.46)
eap	1.87 (.95)	1.73 (.58)	0.14 (.57)	1.54 (.66)	-0.41 (.64)	1.97 (.44)	0.01 (.49)
eca	-1.88 (.88)	0.81 (.54)	-2.69 (.53)	0.95 (.59)	-2.08 (.57)	0.01 (.41)	-1.92 (.45)
landav	-0.45 (.26)	-0.47 (.16)	0.02 (.16)	-0.53 (.18)	-0.11 (.17)	-0.16 (.12)	-0.02 (.13)
rulelaw	0.94 (.35)	0.44 (.22)	0.49 (.21)	0.56 (.24)	0.54 (.23)	0.43 (.16)	0.16 (.18)
N	89	89	89	68	68	89	89
F	11.57 (.000)	8.75 (.000)	12.98 (.000)	8.21 (.000)	16.08 (.000)	11.64 (.000)	10.73 (.000)
$\bar{R}^2$	.375	.306	.404	.350	.530	.377	.356

[a] Figures in parentheses are robust standard error estimates for coefficient estimates and p-values for F statistics. The sample size is less than 92 for Equations 9, 10, and 14 because some instruments were not available for all countries.

Appendix 3

First-Stage Regression Estimates – Models in Levels[a]

	Eqn. 12		Eqn. 13	
	$Y^{+}_{c,t}$	$Y^{-}_{c,t}$	$Y^{+}_{c,t}$	$Y^{-}_{c,t}$
constant	7.66 (.85)	0.44 (.77)	0.32 (.08)	-0.01 (.06)
agroprodav	-0.22 (1.04)	0.97 (.93)	-0.05 (.03)	-0.06 (.02)
eap	0.82 (.73)	-0.77 (.66)	0.12 (.02)	-0.00 (.02)
eca	-3.96 (.84)	3.27 (.75)	0.04 (.03)	-0.06 (.02)
$Y^{p}_{c,t-5}$			0.02 (.03)	-0.01 (.02)
$Y^{+}_{c,t-5}$			0.96 (.03)	0.01 (.02)
$Y^{-}_{c,t-5}$			-0.06 (.03)	0.99 (.02)
N	118	118	118	118
F	10.90 (.000)	10.96 (.000)	35413 (.000)	45667 (.000)
$\bar{R}^2$	.202	.204	.999	.999

[a] Figures in parentheses are robust standard error estimates for coefficient estimates and p-values for F statistics. The variables agroprodav and eap were retained in the first-stage regressions for Equation 12 so as to make the instruments used for these two equations more similar. The first-stage regressions for Equation 13 fit so well because $Y^{+}_{c,t}$ and $Y^{-}_{c,t}$, like most aggregate income series, are highly autocorrelated.